Journal
of
The Franklin Institute

EDITOR, HENRY BUTLER ALLEN, MET.E., D.SC., SC.D., D.ENG.
ASSISTANT EDITOR, NANCY S. GLENN, M.A.

Associate Editors:

WILDER D. BANCROFT, PH.D. — PAUL D. FOOTE, PH.D. — W. F. G. SWANN, D.SC.
CHARLES B. BAZZONI, PH.D. — C. E. K. MEES, D.SC. — HUGH S. TAYLOR, D.SC.
ARTHUR L. DAY, SC.D. — WILLIAM B. MELDRUM, PH.D. — A. F. ZAHM, PH.D.
R. EKSERGIAN, PH.D. — HENRY C. SHERMAN, SC.D. — JOHN ZELENY, PH.D.

Committee on Publications:

CHARLES B. BAZZONI, CHAIRMAN — RICHARD HOWSON — HIRAM S. LUKENS
G. H. CLAMER — CLARENCE L. JORDAN — WINTHROP R. WRIGHT
LIONEL F. LEVY

Vol. 250 **AUGUST, 1950** **No. 2**

CONTENTS

Walter Rogers Johnson ..... GEORGE E. PETTENGILL 93

Differential Analyzer Solution for the Stresses in a Rotating Bell-Shaped Shell ..... J. L. MERIAM .. 115

Radiation Transfer Considerations in the Heating of a Cathode Sleeve ..... W. E. DANFORTH AND T. A. HADDAD 135

Radiation Heating of a Plane Parallel Slab by a Parallel Semi-Infinite Heater ..... W. E. DANFORTH 146

Interflections in Coupled Enclosures ... PARRY MOON AND DOMINA EBERLE SPENCER 151

Notes from the National Bureau of Standards 167

The Franklin Institute
- Museum 173
- Journal of The Franklin Institute 173
- Form of Bequest 174
- The Franklin Institute Laboratories 174
- Library 175

Notes from the Biochemical Research Foundation 179

Book Reviews 181

Book Notes 188

Current Topics 114, 134, 190

Published at
Prince and Lemon Streets, Lancaster, Penna., by
THE FRANKLIN INSTITUTE OF THE STATE OF PENNSYLVANIA
Benjamin Franklin Parkway at Twentieth St., Philadelphia 3, Penna.

DOMESTIC—EIGHT DOLLARS PER YEAR **FOREIGN—NINE DOLLARS PER YEAR**
(Foreign Postage Additional)
SINGLE CURRENT NUMBERS—ONE DOLLAR EACH
REPRINTS—FIFTY CENTS PER INDIVIDUAL INSTALLMENT

Indexes to the semi-annual volumes of the JOURNAL are published with the June and December numbers

THE FRANKLIN INSTITUTE OF THE STATE OF PENNSYLVANIA

welcomes as members all those interested in its purposes and its activities

ANNUAL MEMBERS

Sustaining	$50.00
Active Family	20.00
Active	15.00
Active Non-Resident (50 miles or more from Philadelphia)	7.50
Associate Family	10.00
Associate	5.00
Student (under 25), with Library privileges	3.00
Student (under 25), without Library privileges	2.00

LIFE MEMBERS

Active	$300.00
Active Non-Resident (50 miles or more from Philadelphia)	100.00
Associate	100.00

PRIVILEGES

Free admission to the Museum, Planetarium, and Institute Lectures is granted to all members and to the families of Sustaining, Active Family, and Associate Family members.

The Institute News, which includes news items about the Institute as well as announcements of meetings and lectures, is sent to all members.

The *Journal of The Franklin Institute* is sent to Sustaining, Active Life, Active and Active Non-Resident members.

Use of the Library is granted to Sustaining, Active Family, Active, and Active Non-Resident, as well as to the $3.00 Student members.

THE FRANKLIN INSTITUTE **Benjamin Franklin Parkway, Philadelphia 3, Pa.**

Gentlemen: I desire to contribute to the work of The Franklin Institute by enrolling as ..... Member, for which I enclose payment of $..... the amount due per annum.

NAME .....
(Please print)

ADDRESS .....

.....

.....

SIGNATURE .....

Membership contributions are deductible for income tax purposes.

Journal
of
The Franklin Institute
Devoted to Science and the Mechanic Arts

Vol. 250 AUGUST, 1950 No. 2

WALTER ROGERS JOHNSON

BY

GEORGE E. PETTENGILL[1]

Among the American scientists of the first half of the nineteenth century who played a supporting role to the leaders of science like Joseph Henry and Benjamin Silliman, Walter Rogers Johnson is worthy of note. No examplar of the brilliant deduction, he was a methodical investigator and careful experimenter, catholic in his interests, whose work generally had a practical aim in view. Johnson was associated with The Franklin Institute for ten years, and it is perhaps fitting that he should be studied in the JOURNAL to which he contributed many of his papers. The *raison d'être* for the present paper is the presentation of a bibliography of his writings, not before listed in their entirety. At the same time an opportunity has been taken to introduce certain facts not noted by earlier biographers.[2]

Walter Rogers Johnson was born June 21, 1794 in Leominster, Massachusetts, the son of Luke Johnson, a farmer, who had been active in the American Revolution. The youngest of three children and the only son, Johnson displayed early in life a fondness for learning and a

[1] Assistant Librarian, The Franklin Institute, Philadelphia, Pa.

[2] The following articles have been located about Johnson:

[Mrs. Walter Rogers Johnson], "Walter R. Johnson," *American Journal of Education*, 5: 781–796, 1858. Reprinted in Henry Barnard, "Educational Biography." 2nd ed. New York, F. C. Brownell, 1861. 1: 281–296.

This article is unsigned and has been credited by later writers to Barnard. However, William W. Johnson in "Genealogy. Records of the Descendants of David Johnson of Leominster," Milwaukee, 1876, p. 13, says Mrs. Johnson wrote it, nor does Barnard claim it in the index to his magazine, although he does so for other unsigned articles.

S. W. PARR, "A Pioneer Investigator," *Industrial and Engineering Chemistry*, 18: 94–98, January, 1926.

JOSEPH J. MCCADDEN, "Walter Rogers Johnson, A New England Founder of Free Schools in Pennsylvania," *New England Quarterly*, 11: 121–145, March, 1938.

(Note—The Franklin Institute is not responsible for the statements and opinions advanced by contributors in the JOURNAL.)

taste for books. This stood him in good stead, for while continuing his own studies, he taught in a local district school. After attending Groton Academy one year, he entered Harvard University at the age of twenty-one, receiving his degree in 1819. Forced by financial circumstances to forego studying for a chosen legal career, he became a preceptor in Framingham Academy and in 1821 he went to the Classical School at Salem, Massachusetts. Here he became identified with the cultural and social activities which Salem offered at the time and found pleasure in his acquaintance with the scientific men of the community, including the venerable Nathaniel Bowditch. Mrs. Johnson relates that at about this period her husband turned his hand to writing poetry, but little seems to have survived in printed form.

In November 1821, the principal of Germantown Academy in Pennsylvania, J. M. Brewer, a Harvard graduate, resigned and recommended Johnson as his successor. The trustees at once employed him, thus bringing to the Philadelphia area a man who was to have considerable influence on the local educational scene. He remained as principal at the Germantown Academy for nearly five years and had evidently a likable and successful personality. One Academy historian records that the six years of Brewer's and Johnson's administrations were the brightest in the history of the school from its beginning in 1760.[3]

Among Johnson's pupils at the Academy were Fernando Bolivar, nephew and adopted son of the Liberator of South America, and Robert Montgomery Bird, later a distinguished dramatist and novelist, and a fellow faculty member of Johnson at the Pennsylvania Medical College.[4] A memorable event during his principalship was a visit which General Lafayette paid to Germantown Academy on July 20, 1825. Johnson rose to the occasion and delivered a welcome to the famous visitor which was preserved for posterity in the daily press.[5] Oratory was a favorite means of expression for Johnson and a year later he delivered a speech to commemorate the virtues and services of Thomas Jefferson and John Adams, who had recently died.

During this period Johnson began his efforts in aid of public education in Pennsylvania. He had early become convinced that education was the only sure foundation for a safe and prosperous republic. To help attain this goal he devoted much of his time to propaganda activities, traveling throughout the state.

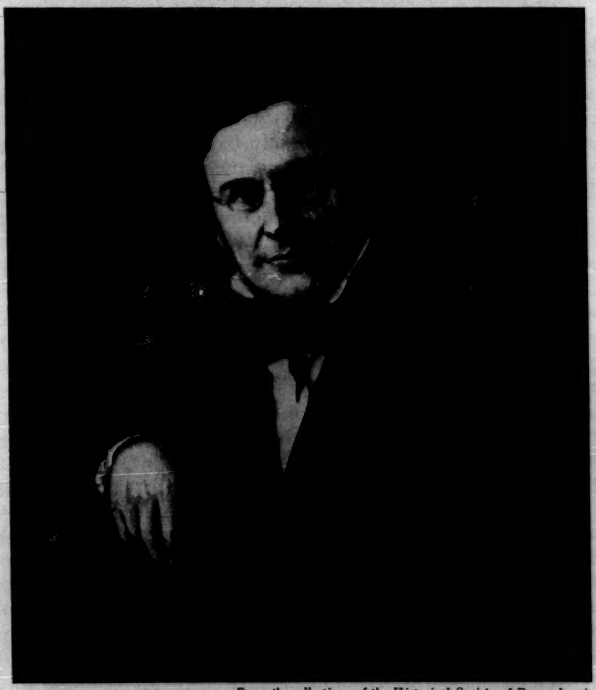
Soon after he came to the Germantown Academy he published in

[3] Germantown Academy, "A History of the Germantown Academy. Published upon the One Hundred and Fiftieth Anniversary of the School's Founding, 1910." [Philadelphia, 1910], p. 155.

[4] CLEMENT E. FAUST, "The Life and Dramatic Works of Robert Montgomery Bird." New York, Knickerbocker Press, 1919. On pages 22–23 there is reprinted the certificate given to Bird by Johnson, which conveys a good idea of the scope of the curriculum of the period.

[5] See bibliography, item 3.

The Commonwealth, a Harrisburg paper, a series of thirteen essays on education, which included suggestions for establishing a system of common schools in Pennsylvania. In 1825 he wrote on the improvement of teacher-training institutions in the United States and followed this with an article on Pennsylvania laws for the support of public education, covering the period of both the province and the state. Articles on other educational problems flowed from his pen for several years and



From the collections of the Historical Society of Pennsylvania

he contributed an account of education in the United States for the American edition of the *Edinburgh Encyclopedia* published in 1832.

Johnson also took an active role in the educational associations of his day, among which was the Pennsylvania Society for the Promotion of Public Schools. He was elected a member of its council in 1829 and carried a major part of the burden thereafter. It has been pointed out that Johnson was apparently the only teacher who was a member of the organization, an indication of the rather poor standing of the rank and

file of teachers at that period.[6] Johnson was also a member and director of the Philadelphia Asspciation for Teachers, one of whose aims was to awaken public attention to educational needs.

How much influence all these various promotional activities had in securing the final result of a free public school system in Pennsylvania it is difficult to determine. Perhaps it cannot be better expressed than Mr. McCadden has done: "It cannot, however, be doubted that his persistent, prolific, far-sighted, and scholarly devotion to the cause of public education in Pennsylvania helped materially in lifting that cause to victory."[7]

These promotional activities, begun during Johnson's administration at Germantown Academy, continued after he became associated with The Franklin Institute High School in 1826. This phase of Johnson's career has been inadequately treated and even the histories of the Institute have little to say about the school.[8]

The Board of Managers of The Franklin Institute on February 2, 1826 directed its Committee of Instruction to inquire into the expediency of extending the schools of the Institute so as to embrace the different branches of elementary education. Johnson was consulted by the Committee soon after it began its discussions and he outlined the subjects to be studied, suggested the use of the monitorial system, discussed the possible use of premiums as awards, and expressed the thought that the terms could not reasonably be put less than seven dollars per quarter. There was considerable discussion by the Board of Managers since proposals had been put forth at the same time that there should be established in Philadelphia a polytechnic college for which its promoters hoped to secure the backing of The Franklin Institute. However, this latter proposal failed to win sufficient support and the Board voted on April 19, 1826, to establish a High School "in which the system of mutual instruction shall be introduced." Johnson was elected principal on May 3 and rooms on the third floor of the Institute building on South Seventh Street were appropriated for the use of the school. Preparations went on apace and Johnson took the opportunity in view of his forthcoming association with the Institute to seek membership in it.

The school opened in the fall of 1826 with about three hundred pupils and a faculty of four teachers besides Johnson. The exact terms of the agreement between the Institute and Johnson have not been located. It would seem, however, that Johnson collected the tuition,

[6] JOSEPH J. MCCADDEN, "Roberts Vaux and His Associates in the Pennsylvania Society for the Promotion of Public Schools," *Pennsylvania History*, 3: 1–17 January, 1936, p. 10.

[7] MCCADDEN, *New England Quarterly*, 11: 145, 1938.

[8] It is not the writer's purpose to go into detail here as he hopes to prepare a comprehensive history of the school based on the existing manuscript records. However an outline of the principal facts, many previously unrecorded, will be related here.

which had been set at the recommended seven dollars per quarter, and paid rent for the rooms. The Committee of Instruction nonetheless was responsible for the general supervision of the school. That the members took their duties seriously is shown in the report of March 1828, which states that the Committee had arranged itself into several committees of supervision charged with visiting the school at least once each day.

For three years the Institute's arrangement with Johnson continued. Then on August 13, 1829 the Committee of Instruction reported: "The third year of the school terminated with this examination and agreeably to arrangements made with Mr. W. R. Johnson by the late committee appointed by the Board, the duties of the High School of The Franklin Institute have ceased & will therefore no longer be under the control of your committee." Johnson continued the school for another three years in the Institute building, and then he moved it to other rooms on St. James Street near Seventh Street where he continued it three or four years longer.

While conducting the High School, Johnson prepared two textbooks for the use of his students and encouraged his teachers in producing other manuals. From July to December 1828 there appeared in the JOURNAL OF THE FRANKLIN INSTITUTE a series of six articles by Johnson entitled "On the combination of a practical with a liberal course of education," which comprised essentially a description of the High School, outlining its aims, describing its equipment and discussing the operation of the monitorial system.

In addition to his principalship of the High School, Johnson soon assumed other duties at the Institute, for on October 23, 1828 he was appointed to deliver a course of lectures on Mechanics and Natural Philosophy. These lecture courses had been a regular feature of the Institute's educational program ever since it opened in 1824 and were completely independent of the various schools run by the Institute. The men giving the courses were generally denominated Professor, and the lack of this title apparently caused Johnson some reluctance in accepting. The next year, however, all was made right when he was elected Professor of Mechanics and Natural Philosophy. With the 1831–32 season he was relieved of the mechanics part of his duties thus allowing him to devote more time to the natural philosophy part of his professorship. In reporting the change the Board of Managers emphasized the importance of natural philosophy naming it "the very element of all the subjects taught in the Institute's lecture room." Johnson continued in this professorship until, expecting to leave Philadelphia, he resigned it early in 1837.

His approach to these lectures was outlined in a communication to a sub-committee of the Committee of Instruction July 3, 1833, five years after he began lecturing when he wrote:

> Keeping in sight the obvious and often declared truth, that the institution is indebted for its chief usefulness to its courses of scientific instruction, and, that for reasons not limited to the mere hope of pecuniary advantage, the mechanic ought to enjoy opportunities of making solid attainments both in the theory and practice of his art, I have endeavoured so to exhibit theory that practice should not be forgotten, & so to illustrate practice, that theory should not be dishonoured. We ought not on the one hand to attempt to deal in useless abstractions nor on the other to degrade science into a mere exhibition of frivolous experiments or the juggling tricks of legerdemain. There should be substantial scientific truth imparted for the benefit of those (and they are many) who before attending the lectures are not fully informed in the principles of science, & there should be interesting practice exhibited and explained to the comprehension of all. We ought not to expect the permanent usefulness or respectability of the institution, to rest on any other basis than the preservation of a high standard of *practical science*. The application of science to the arts, has ever been a favorite object of study with me, & will so continue.

After his association with The Franklin Institute and especially after his appointment as professor, Johnson turned his attention more and more to scientific matters to the gradual exclusion of educational problems. His first scientific paper appeared in the JOURNAL OF THE FRANKLIN INSTITUTE in May, 1829, and was followed by others in the same journal, in the *American Journal of Science and Arts*, and in the various publications of the Academy of Natural Sciences of Philadelphia. His first major scientific opportunity came when, as a member of the Institute's Committee on the Explosions of Steam-boilers, he was made chairman of the Sub-committee on the Strength of Materials.

This sub-committee, appointed January 4, 1831, began its experiments on April 4 of the next year after lengthy preparations and conducted the last test on January 5, 1837, some five years later. In connection with the tests it was necessary to build a testing machine, and this machine, probably the first such built in America,[9] is still a treasured possession of The Franklin Institute. The experiments were detailed and involved hundreds of tests on cast iron, wrought iron, copper and cast steel. The effect of temperature on the tensile strength of the metals was also investigated which required ingenious arrangements for measuring temperatures. Above the boiling point of mercury, about 675° F., it was necessary to use a steam pyrometer, which involved extensive tests to determine specific heats and the latent heat of water vapor. The report was published both in the JOURNAL OF THE FRANKLIN INSTITUTE and as a separate book, and it exerted a considerable influence on the choice of boiler materials. Since the research was

[9] C. H. GIBBONS, "Materials Testing Machines." Pittsburgh, Pa., 1935. Pp. 16–17 give a resume of the use of this machine.

financed in part with federal funds, it marked the first of Johnson's several investigations for the United States government.

In the summer of 1836 Johnson began the geological investigations which were to occupy a large part of the remainder of his life. In December of the same year he was appointed to the scientific corps of the South Sea Scientific and Exploring Expedition. His appointment caused him to resign his professorship in the Institute, but he found that he had little to do. For numerous reasons the expedition was delayed and all he had to do was to secure some scientific supplies. Eventually he did not go on the expedition. There was some question about his being paid for the time he had spent waiting and it is from the considerable correspondence preserved in the report of the Secretary of the Navy concerning the delay in the sailing of the expedition that we learn of his activities. Although a Congressional Committee recommended payment of part of his claim, no record has been found that Congress ever approved the payment.

In 1838 Johnson prepared a memorial to Congress praying for the establishment of a National Institution for the prosecution of experiments and researches in those physical sciences which are required by the public service and for the general welfare of the country. This is the most widely printed of Johnson's works, for it appeared in various forms and at different dates no less than five times. It is of particular significance in view of the subsequent foundation of the Smithsonian Institution.

When a new medical college was established at Philadelphia in 1839, Johnson was chosen Professor of Chemistry and Natural Philosophy. Known as the Pennsylvania Medical College, it functioned as a branch of Pennsylvania College at Gettysburg. Johnson continued in this position until 1843 when the entire faculty resigned as the result of a disagreement. It was during the last two years of his tenure that his former Germantown Academy student, Robert M. Bird, served as a colleague on the faculty.

As evidence of Johnson's inquiring mind one may note his early attention to the making of daguerrotypes. He was among the pioneers of the art in Philadelphia and on January 31, 1840[10] he advertised a lecture on the subject to be given the following evening. This was to be illustrated with Daguerre's complete apparatus and numerous examples taken in the city. Although Johnson apparently never published anything constructive on the subject, we do know that he had acquired some skill, for one journalist reports having seen "an exquisite view of the Exchange, being one of many perfect specimens of the art obtained by Professor W. R. Johnson of this city."[11]

While engaged in his teaching at the Medical College, Johnson was

[10] *National Gazette and Literary Register*, Jan. 31, 1840, p. 2; also *United States Gazette*, Jan. 31, 1840, p. 3.

[11] *National Gazette and Literary Register*, Feb. 7, 1840, p. 2.

also pursuing other free lance activities. He was beginning to make wide-spread investigations of coal and iron fields, many times as a consultant, a common occupation of early nineteenth century American scientists. He also carried on numerous researches for the government, especially for the Navy Department, on such subjects as floating docks, inventions to prevent the explosion of steam boilers, sheathing copper, cables, and thermotension.

Johnson's interest in geology and his work in the coal fields led from evaluating coal resources to a consideration of the uses of coal, and during the latter years of his life, he devoted a great deal of effort to coal investigations. In 1841 he published a book on the use of anthracite in the manufacture of iron. At about this time he realized the need for a scientific investigation of the properties of all the varieties of coal in order to ascertain their absolute and relative values as fuels. He proposed this research to the Secretary of the Navy. In 1841 Congress passed a law which authorized making experiments in matters connected with the naval service and the national defense. Johnson was soon at work and tested some forty-one varieties of coal. His primary object was to ascertain their evaporative power but he also made analyses and conducted other experiments. His experimental data and conclusions formed a volume of about 600 pages. This was an outstanding piece of research and gave practical data by which the Navy could judge the coal best suited to its needs. Dr. Parr's comment is of interest: "One cannot go through the pages of this report without marveling at the indefatigable energy that must have characterized the author, the skill with which he developed a technic along lines having no established precedents to follow and the wisdom with which he arranged and interpreted the results."[12] If imitation is the sincerest flattery, one should note that the British Admiralty and the Prussian government soon undertook their own researches primarily employing Johnson's basic techniques, for which the Prussian investigator, W. Brix, generously acknowledged his indebtedness.[13]

Much of his later work on coal may be found in scattered minor references in the *Proceedings* of the Academy of Natural Sciences, which were afterwards gathered together to form part of his volume "The

[12] PARR, *op. cit.*, p. 95.

[13] HENRY THOMAS DE LA BECHE AND LYON PLAYFAIR, "First (to Third) Report on the Coals Suited to the Steam Navy." London, Her Majesty's Stationery Office, 1848–1851. Also published in Parliamentary Papers. The first report was reprinted in part by Johnson in item 120. PHILIPP WILHELM BRIX, "Untersuchungen über die Heizkraft der wichtigeren Brennstoffe des Preussischen Staates." Berlin, Ernst & Korn, 1853.

Johnson's report and these two foreign reports are discussed at some length in "Steinkohlen," *Wagner's Jahres-Bericht über die Fortschritte und Leistungen der Chemischen Technologie und Technischen Chemie*, 6: 623–658, 1860, in connection with a report by E. Hartig on coals of Saxony.

In the Library of Congress copy of Brix's work there is bound a complimentary letter to Johnson from the author.

Coal Trade of British America," 1850. In 1851 Johnson went to Europe, spending much of his eight months' stay in the inspection of various coal fields.

Several instances in which Johnson was a pioneer worker in the field have already been noted. Yet another may be mentioned. In 1845 he made a report in collaboration with John B. Jervis on a water supply for Boston. Considerable attention was paid by the engineers to the problem of animalcules in the water. Their report and an appendix by J. W. Bailey have been called "the earliest notable studies of algae and related troublesome organisms in public water supplies made in America."[14]

In addition to Johnson's activities in The Franklin Institute, he played a prominent role in other leading scientific associations. He was elected a member of the Academy of Natural Sciences of Philadelphia, February 27, 1827 and during the next twenty years served it in several capacities—auditor, 1832–1835; curator, 1835–1837; member of the publication committee, 1836–37; and corresponding secretary, 1841–1848. He was further honored when in 1839 he was selected to deliver the address on the laying of the cornerstone of the Academy's new building.

Johnson was one of the founding members of "The Association of American Geologists" organized in the hall of The Franklin Institute on April 2, 1840. When in September 1847 it was voted to resolve the then "Association of American Geologists and Naturalists" into the "American Association for the Advancement of Science" he was elected the first secretary and subsequently served on the Standing Committee for 1848–49. He joined the National Institute on its first founding as a corresponding member, and later after he removed to Washington he served as corresponding secretary. The Maryland Institute for the Promotion of Mechanic Arts bestowed an honorary membership on him and he delivered lecture series before it during the years 1849 to 1852.

During the later years of his life Johnson held no formal position but carried on numerous activities. He edited the American editions of two standard foreign works on mechanics and chemical technology and continued his geological surveys. In 1848 he moved to Washington in the hope that the government would continue the coal investigations, an unfulfilled ambition.[15]

---

[14] M. N. BAKER, "The Quest for Pure Water," New York, American Water Works Association, 1948, p. 396.

[15] Some sources say that Johnson was employed as a chemist at the Smithsonian Institution from 1848 to 1852. Although he did apply for a position with the Smithsonian, the writer has found no other corroboration of his having been so employed, nor does his wife mention it. He was apparently conducting some experiments there shortly before his death, and it is probable that the facilities were lent him in connection with the building stone investigation mentioned below.

In 1851 Johnson was chosen one of twenty-one members of the central committee appointed by the National Institute, at the request of the government, to serve as American Commissioners to promote the Exhibition of 1851 in London. He served as secretary of the group and embraced the opportunity it offered for a trip overseas, during which he visited England, Wales, Germany, Switzerland, Italy and France.

Soon after his return Johnson again found his services in demand by the government. In January 1852 he was summoned to testify before a House of Representatives Committee upon the building stone used in constructing the foundation of the extension of the United States Capitol. His report two months later represented his last piece of work for he died shortly thereafter on April 26, 1852, after a brief illness of six days.

By most standards Walter Rogers Johnson was a successful man. He achieved a career for himself in two fields, education and science. He was a pioneer in several fields and his investigations of coal were of the first importance. His wife writes in her biography of him that he was generally well paid for his professional services so that he was able to live in respectable independence and generous hospitality. In character he appears to have been a man of moderation and temperance with a humane outlook on life. Probably he was not a great man, but typical of those who were busy developing American science during the early nineteenth century and contributing to the nation's industrial development.

BIBLIOGRAPHY OF THE WRITINGS OF WALTER ROGERS JOHNSON

This bibliography is intended to comprise a complete listing of Johnson's writings in book, pamphlet and periodical form, with the addition of such items as have been located in newspapers. It is hoped that it is complete in the three major categories, but no attempt has been made to examine exhaustively the newspapers to which Johnson might have contributed. Considering the frequency with which his writings were reprinted and excerpted, it is difficult to be certain that every such instance has been located, especially in foreign periodicals.

The items have been arranged chronologically. It should be noted that many of the references in the *Proceedings* of the Academy of Natural Sciences of Philadelphia are little more than abstracts of papers read before its meetings. However, since some had been noted in bibliographical sources, it seemed better to include all, than to attempt to be selective.

An asterisk (*) has been used to indicate that a reprint or separate of a particular article exists.

1. [Poem on leaving New England] in William W. Johnson, *Genealogy. Record of the Descendants of David Johnson of Leominster.* Milwaukee, Godfrey & Crandall, 1876, p. 13.

2. Essays (thirteen) on education, with suggestions for establishing a system of common schools in Pennsylvania. Published in the "Commonwealth." Harrisburg, 1822–23.

This entry copied from bibliography appended to Mrs. Johnson's biography of Johnson in *American Journal of Education* 5: 797, 1858, since the copies of the *Commonwealth* containing these articles could not be located.

3. [Address on the occasion of the visit of General Lafayette to Germantown Academy July 20, 1825.] *Poulson's American Daily Advertiser* (Phila.), July 26, 1825, p. 2.
Reprinted in Charles F. Jenkins, *Lafayette's Visit to Germantown, July 20, 1825*. Philadelphia, 1911, pp. 43–46.

4. Observations on the improvement of seminaries of learning in the United States: with suggestions for its accomplishment. Philadelphia, E. Littell, 1825. 28 p.
Reviewed in *U. S. Literary Gazette* 3: 171–183, 218–221, Dec. 1, 15, 1825, and in *Am. J. Educ.* 1: 677–688, Nov., 1826. "Schools for teachers" was reprinted from this in *Am. J. Educ.* 5: 799–802, 1858.

5. [Review of] 1. An act of the General Assembly of the Commonwealth of Pennsylvania, to provide more effectually for the education of the poor gratis, and for laying the foundation of a general system of education throughout this Commonwealth. Passed March 29, 1824. Harrisburg, 1824. 2. Remarks on the school law of the last session of the legislature, and information concerning the common schools of Massachusetts, New York, South Carolina, &c. &c. Philadelphia, 1826. 8vo. pp. 38. 3. Eighth annual report of the Controllers of the public schools of the First school district of the State of Pennsylvania; with their accounts. Philadelphia, 1826. 8vo. pp. 15. *U. S. Literary Gazette* 4: 121–133, May 15, 1826.
Reprinted under the following title: *"Remarks on the legal provisions for education in Pennsylvania. Remarks on the provisions made by law, for the support of education in Pennsylvania, from the first settlement of the Province in 1682, to the year 1826." *Hazard's Register of Pennsylvania* 4: 293–297, Nov. 7, 1829. Reviewed with extracts in *Am. Annals of Educ.* 1: 358–366, Aug., 1831.

6. An oration delivered at Germantown, Pennsylvania, on the 20th July, 1826, in presence of the citizens of Germantown, Roxborough, Bristol, and Penn townships, assembled to commemorate the virtues and services of Thomas Jefferson and John Adams. Philadelphia, Robert H. Small, 1826. 25 p.
Reprinted in Charles F. Jenkins, *Jefferson's Germantown Letters together with other papers relating to his stay in Germantown during the month of November 1793*. Philadelphia, William J. Campbell, 1906, pp. 171–194.

7. * On the combination of a practical with a liberal course of education. JOURNAL FRANKLIN INSTITUTE 6: 55–57, 108–113, 166–169, 275–278, 353–355, 367–369, July–Dec., 1828.
Also *Am. J. Educ.* 3: 600–609, 733–736, 1828.

8. Address introductory to a course of lectures on mechanics and natural philosophy, delivered before The Franklin Institute, Philadelphia. Boston, Hiram Tupper, 1829. 15 p.
Also *Am. J. Educ.* 4: 23–26, Jan.–Feb., 1829.

9. Observations on an apparatus to illustrate the composition and resolution of forces, and to measure the power obtained by certain parts of machinery operating on this principle. JOURNAL FRANKLIN INSTITUTE 7: 354–358, May, 1829.

10. An introduction to the Greek language; together with a key to facilitate the literal and free translation of the text, to point out the grammatical construction of sentences, to show the inversions of style, to supply ellipses, and to explain idiomatical expressions. Boston, Hilliard, Gray, Little, and Wilkins. Philadelphia, Towar and Hogan, 1829. iv, 93 p.
With this is bound: The text of Johnson's Introduction to the Greek language; to which is adapted a key to facilitate the literal and free translation of the text, to point out the grammatical construction of sentences, to show the inversions of style, to supply ellipses, and to explain idiomatical expressions. Boston, Hilliard, Gray, Little, and Wilkins. Philadelphia, Towar and Hogan, 1829. ii, 27 p. A review with the incorrect title "An introduction to the French language" appeared in *Am. J. Educ.* 4: 566–567, Nov.–Dec., 1829.

11. First lessons in practical geometry. Containing such problems as are essentially necessary in exercising most of the industrious professions. Intended for the use of students in elemen-

tary schools. Translated from the French of L. Gaultier . . . Philadelphia, Towar & Hogan; Boston, Hilliard, Gray & Co., 1829. 54 p.

Reviewed in *Am. J. Educ.* n.s. 1: 435, Sept., 1830.

12. * On the relation between rolling and dragging friction. Read before The Franklin Institute, at the monthly meeting, October 22, 1829. JOURNAL FRANKLIN INSTITUTE 9: 57–69, Jan., 1830.

Separate has title: "Observations on the relation . . ."

13. * Observations on the electrical characters of caoutchouc, or gum elastic; with some applications of which they are susceptible. *J. Acad. Nat. Sci. Phila.* 6: 245–252, 1830.

Summarized and quoted in part in *Am. J. Sci. Arts* 20: 404–406, July, 1831.

14. Remarks on the duty of the several states, in regard to public education. Philadelphia, Printed for the author, by William Sharpless, 1830. 8 p.

15. Observations and experiments on the rapid production of steam in contact with metals at a high temperature. *Am. J. Sci. Arts* 19: 292–299; 20: 308–317, 418, Jan., July, 1831.

Abstracted in Royal Institution. *Journal* 1: 613–614, 1831; 2: 393–395, 1831. *Mechanics' Magazine* (London) 17: 26–27, April 14, 1832. *Dingler's Polytechnisches Journal* 44: 439–442, June, 1832. As a separate has the following title: "Experimental inquiries respecting heat and vapor; with some practical applications." n.p., n.d. 18 p. But compare item 21.

16. Description of an apparatus, called the Rotascope, for exhibiting several phenomena, and illustrating certain laws of rotary motion. JOURNAL FRANKLIN INSTITUTE 12: 361–366, Dec., 1831.

A fuller version appears in * *Am. J. Sci. Arts* 21: 265–280, Jan., 1832. In *Am. J. Sci. Arts* 2d ser. 21: 146, Jan., 1856, a note entitled "Rotascope of Prof. Walter R. Johnson" refers to publication of a notice of the instrument in *L'Institut* (Paris) Nov. 15, 1855, and comments on it briefly.

17. A lecture on the importance of linear drawing, and on the methods of teaching the art in common schools and other seminaries. Delivered in the Representatives' Hall, Boston, August 23, 1830, before the American Institute of Instruction. Boston, Hilliard, Gray, Little and Wilkins, 1831. 19 p.

Also issued in: American Institute of Instruction. *Introductory discourse and lectures*, August, 1830. Boston, 1831, pp. 257–275.

18. Remarks on the nature and importance of enlarged education, in view of the present state of society in Europe and America. n.p. [1831] 8 p.

19. Education in the United States. In *Edinburgh encyclopedia*, conducted by David Brewster. First American edition, corrected and improved. Philadelphia, 1832. v. 18, pp. 353–362.

Separate has title "A concise view of the general state of education in the United States." n.p., n.d. 10 p. Another separate: [Philadelphia, W. F. Geddes, 1832] 9 p. Reprinted under second title in *Hazard's Register of Pennsylvania* 10: 257–265, Oct. 27, 1832.

20. Observations and experiments on the variable rapidity of action, between water and hot iron. *Am. J. Sci. Arts* 21: 71–85, Jan., 1832.

21. Experimental inquiries respecting heat and vapor, with some practical applications. *Am. J. Sci. Arts* 21: 304–314, Jan., 1832.

A reprint exists with this title: n.p., n.d. 54 p. which includes the following items: 15, 20, 21, 22 from *Am. J. Sci. Arts*, v. 19–22. Compare item 15.

Items 15, 20, 21, 22 also appear under this title, with minor omissions, in U. S. House of Representatives. *Report of select committee on explosions of steam boilers.* n.p., 1832. (22d Cong. 1st Sess. House. Report no. 478, pp. 97–140.)

22. Description of an instrument called the steam pyrometer. *Am. J. Sci. Arts* 22: 96–104, July, 1832.

23. Monitorial schools. Remarks on the 'extent to which the monitorial system is advisable and practicable in common schools.' Communicated to the American Lyceum. *Am. Annals of Educ.* 2: 413–420, Aug. 1, 1832.

24. * Remarks on the strength of cylindrical steam boilers. JOURNAL FRANKLIN INSTITUTE 14: 149–154, Sept., 1832.
Reprinted with the addition of one paragraph in **Am. J. Sci. Arts* 23: 68–75, Jan., 1833. Separate from *Am. J. Sci. Arts* contains also item 25. Reprinted in *Repertory of Patent Inventions* 15: 44–53, Jan., 1833 from which it was translated into German in *Dingler's Polytechnisches Journal* 48: 81–88, April, 1833.

25. Notice of an ancient American utensil. *Am. J. Sci. Arts* 23: 65–67, Jan., 1833.

26. * On the economy of fuel with reference to its domestic applications. *Am. J. Sci. Arts* 23: 318–337, Jan., 1833.
Reprinted with title "On the economy of heat" and without illustrations, descriptions and some text, in *Hazard's Register of Pennsylvania* 11: 113–118, Feb. 23, 1833.
Separate from *Am. J. Sci. Arts* has title "On the economy of heat."

27. Chronological view of the enactments on the subject of education. *Hazard's Register of Pennsylvania* 11: 1–6, 17–25, Jan. 5, 12, 1833.

28. Education in Pennsylvania and New York. *Hazard's Register of Pennyslvania* 11: 49–52, Jan. 26, 1833.

29. On the utility of visible illustrations. A lecture prepared for the American Institute of Instruction. *Am. Annals of Educ. and Instruction* 3: 97–112, March, 1833.
Also in American Institute of Instruction. *Introductory discourse and lectures*, August, 1832. Boston, 1833, pp. 65–85.

30. On the Eupatorium Huaco. *Am. J. Sci. Arts* 24: 279–285, July, 1833.
See also item 36, which uses the preferred spelling Guaco.

31. Improved economy of the steam engine. *U. S. Gazette* (Phila.) 71: 2, Sept. 20, 1833.
This is signed only W. R. J. but it seems likely that it is by Johnson. Reprinted in *Hazard's Register of Pennsylvania* 12: 212–213, Oct. 5, 1833.

32. Observations on some experiments in electricity. *Am. J. Sci. Arts* 25: 57–68, Jan., 1834.

33. [A letter to Samuel Breck, dated Dec. 30, 1833, on the subject of common schools, manual labour in schools, and seminaries for teachers] Pennsylvania. Legislature. *Report of the joint committee of the two houses of the Pennsylvania Legislature on the subject of a system of general education* . . . Harrisburg, Henry Welsh, 1834, pp. 51–52.
Also in Pennsylvania. House of Representatives. *Journal* 1833–34, 2: 615–616. Reprinted in *Hazard's Register of Pennsylvania* 13: 412, June 28, 1834.

34. On the specific heats of solids. On the methods of determining and calculating the specific heats of certain solids, with some precautions to be observed in the experiments. JOURNAL FRANKLIN INSTITUTE 18: 306–317, Nov., 1834.
Reprinted, with slight changes, under the sub-title in * *Am. J. Sci. Arts* 27: 267–280, Jan., 1835.

35. On the apparent anomaly observed in the rotation of liquids of different specific gravities when placed upon each other. *Am. J. Sci. Arts* 27: 84–88, Jan., 1835.

36. Confirmatory notice of the medical virtues of Guaco. *Am. J. Sci. Arts* 27: 171, Jan., 1835.
Supplementary to item 30.

37. A memoir of the late Lewis David von Schweinitz, P.D. with a sketch of his scientific labours. Read before the Academy of Natural Sciences of Philadelphia, May 12th, 1835. Philadelphia, William P. Gibbons, 1835. 38 p.
Reprinted in *Hazard's Register of Pennsylvania* 15: 369–376, June 13, 1835.

38. The scientific class-book; or, a familiar introduction to the principles of physical science, for the use of schools and academies, on the basis of Mr. J. M. Moffat . . . With emendations, notes, questions for examination, lists of works for reference, some additional illustrations, and an index. Philadelphia, Edward C. Biddle, 1835–36. 2 v.

For a later edition see items 86 and 87.

39. * On schools of the arts. American Institute of Instruction. *Introductory discourse and lectures*, August, 1835. Boston, 1836, pp. 271–291.

Reprinted in *Waldie's Select Circulating Library*. Part I, 66–70, 1838.

40. The Schoolmaster, and advocate of education for the use of families, teachers, and schools. Edited by J. Frost, assisted by W. R. Johnson, J. M. Keagy, W. Russell, and J. B. Walker. Philadelphia, 1836.

There were no signed contributions by Johnson in the first eleven issues of volume 1 examined.

41. Mr. Samuel Raub's safety apparatus for steam boilers. JOURNAL FRANKLIN INSTITUTE 23: 3–6, Jan., 1837.

Reprinted in *Mechanics' Magazine* 27: 361–363, 1837, from which it was translated into German in *Dingler's Polytechnisches Journal* 66: 248–251, 1837.

42. * Experiments on the adhesion of iron spikes of various forms when driven into different species of timber. JOURNAL FRANKLIN INSTITUTE 23: 281–292, April, 1837.

Also appeared with the omission of some illustrations in * *Am. J. Sci. Arts* 32: 34–45, July, 1837, from which it was reprinted in *Mechanics' Magazine* 27: 281–287, 303–304, July 29, Aug. 5, 1837.

43. * On the fusing point of zinc, and a reference to the relation between the tenacity and the fusibility of the metals in general. *J. Acad. Nat. Sci. Phila.* 7: 200–216, 1837.

44. * Observations on the effects of a remarkable atmospheric current or storm as witnessed on the day following its occurrence. *J. Acad. Nat. Sci. Phila.* 7: 269–281, 1837.

Reprinted, slightly abridged, with following title: "Observations on the effects of a remarkable storm or tornado." *Edinburgh Journal of Natural History*, p. 160, Sept., 1838.

45. Report of the Committee of The Franklin Institute . . . on the explosions of steam boilers of experiments made at the request of the Treasury Department of the United States. Part II containing the report of the sub-committee, to whom was referred the examination of the strength of materials employed in the construction of steam boilers. Philadelphia, Merrihew & Gunn, 1837. 254 p.

The report, presented by Johnson as chairman of the sub-committee, was also signed by Benjamin Reeves. The report was also published in JOURNAL FRANKLIN INSTITUTE 23: 73–109, 157–193, 241–277, 325–361, 409–451; 24: 1–31, 73–113, Feb.–Aug., 1837. From internal evidence the book was apparently finished before completion of publication in the JOURNAL. Reprinted in large part in *Mechanics' Magazine* 30: 18–26, 119–125, 438–446, 459–477, 1838/39; 31: 8–14, 22–29, 1839. See also item 103 for extract.

46. [Letters of W. R. J. to Mahlon Dickerson, Secretary of the Navy, relative to Johnson's participation in the South Sea Exploring Expedition.] in U. S. Secretary of the Navy. *Exploring expedition*. Message from the President of the United States, transmitting the information required by a resolution of the House of Representatives of 7th December last, in relation to the delay in the outfit, &c. for the exploring expedition. Feb. 7, 1838. n.p., Thomas Allen [1838] 630 p. (25th Cong. 2d Sess. House. Ex. doc. no. 147.)

The letters are dated 1836–1837 and appear on the following pages: 10–13, 34–36, 132–133, 163, 238–239, 256–258, 283–284, 366–368, 402, 458–459, 461.

47. Establishment of an institution for experiments in physical sciences. Memorial of Walter R. Johnson of Philadelphia, praying for the establishment of a National Institution for the prosecution of experiments and researches in those physical sciences which are required by the

public service and for the general welfare of the country. n.p., Thomas Allen [1838]. 11 p. (25th Cong. 2d Sess. House. Doc. 396.)

Another edition with title: "Institution for experiments in physical sciences. Memorial praying for the establishment of a National Institution . . ." n.p., n.d. 18 p. Reprinted in *Waldie's Select Circulating Library.* Part II, 108–110, 1838; Johnson's *Memoir on the Scientific Character and Researches of the late James Smithson.* 1844. 36 p. edition, pp. 26–36. (item 93); and William J. Rhees, *The Smithsonian Institution: Documents Relative to its Origin and History.* Washington, 1879. (Smithsonian miscellaneous collections 328), pp. 172–186.

48. * Experiments on two varieties of iron, manufactured at the Adirondack works, directly from the magnetic ore of McIntyre, Essex County, New York. JOURNAL FRANKLIN INSTITUTE 27: 1–10, Jan., 1839.

Reprinted with slight changes in *Am. J. Sci. Arts* 36: 94–105, Jan./April, 1839.

49. * Analysis of some of the minerals found at Karthaus and Three Runs, on the West Branch of the Susquehanna River, Clearfield County, Pennsylvania. Accompanied by a section of the mineral ground. JOURNAL FRANKLIN INSTITUTE 27: 73–80, Feb., 1839.

50. Specification of a patent for imparting to articles of iron and steel an increase of strength. Granted to Walter R. Johnson, city of Philadelphia, June 30th, 1838. Antedated December 30th, 1837. JOURNAL FRANKLIN INSTITUTE 27: 404, June, 1839.

Reprinted nearly in full in *Mechanics' Magazine* 31: 411–412, 1839, from which it was abstracted in *Dingler's Polytechnisches Journal* 74: 155, 1839.

51. * Examination of some of the anthracites found in Sugar Loaf Township, Luzerne County, Pennsylvania. JOURNAL FRANKLIN INSTITUTE 28: 73–77, Aug., 1839.

52. * Specification of a patent for an improvement in the art of increasing the strength of wrought iron and steel. Granted to Walter R. Johnson, of the city of Philadelphia, July 9th, 1838. JOURNAL FRANKLIN INSTITUTE 28: 232–236, Oct., 1839.

53. * Analysis of some of the anthracites and iron ores found on the headwaters of Beaver Creek; in the counties of Luzerne, Northampton and Schuylkill, Pennsylvania. JOURNAL FRANKLIN INSTITUTE 28: 289–298, Nov., 1839.

A summary with title: "Professor Johnson's analyses of anthracite and iron ore," in *Am. J. Sci. Arts* 38: 382–385, Jan., 1840.

54. Address delivered on laying the corner stone of the Academy of Natural Sciences of Philadelphia; May 25th, 1839. Philadelphia, T. K. & P. G. Collins, 1839. 8 p.

55. Report of an examination of the mines, iron works, and other property belonging to the Clearfield Coke and Iron Company, together with some examinations of the minerals employed in the manufacture of iron found at Karthaus and Three Runs, on the West Branch of the Susquehanna River: accompanied by a section of the mineral ground. Philadelphia, Lydia R. Bailey, 1839. 22 p.

56. [A letter on Daguerrotype.] *National Gazette and Literary Register* (Phila.), Feb. 8, 1840, p. 2, col. 3.

57. Report of a geological, mineralogical, and topographical examination of the coalfield of Carbon Creek, the property of the Towanda Rail Road and Coal Company, Bradford County, Pa., with an analysis of the minerals. Accompanied by a map of the surveys, profile of the road, and sections of the mineral ground. Philadelphia, John C. Clark, 1840. 47 p.

* Extracts, with remarks by the editor, in *Am. J. Sci. Arts* 39: 137–149, April, 1840. The coal analyses were reprinted with the title: "Prof. R. Johnson's report of examinations made five years later than the preceding," in North Branch Iron and Coal Company. *Its lands and avenue to market with reports of surveys and explorations by Messrs. Richard C. Taylor, Walter R. Johnson, and J. P. and Joseph Lesley, Jr.* Philadelphia, James B. Smith & Co., 1855, pp. 22–31.

58. * Description of a specimen of engraving, by the aboriginal inhabitants of North America. With a notice of some incidents in the history of the early settlers on the West Branch of the Susquehanna River. Historical Society of Pennsylvania. *Memoirs*. 4, pt. 1: 93–103, 1840.

59. A lecture, introductory to a course on chemistry and natural philosophy, in the medical department of Pennsylvania College: delivered November 3, 1840. Philadelphia, L. R. Bailey, 1840. 24 p.

60. Notes on the use of anthracite in the manufacture of iron. With some remarks on its evaporating power. Boston, Charles C. Little and James Brown, 1841. vi, 156 p.

A critical notice appeared in *North American Review* 54: 243–244, Jan., 1842. Extracts appeared under the title: "Progress of the anthracite iron manufacture in America," *Mechanics Magazine* 37: 381–384, Oct. 15, 1842.

61. Report of a survey and exploration of the coal and iron lands, belonging to the Allegheny Coal Company, in Somerset County, Pennsylvania, accompanied by maps, profiles, and sections. Philadelphia, Joseph and William Kite, 1841. 64 p.

Johnson's report occupies pp. 3–52, the remainder being an appendix of miscellaneous material. Although an actual comparison has not been feasible this is without much question the report by Johnson on pages 7–37 of the following: Charter of the Somerset Iron and Coal Company of Pennsylvania, with a description of the iron and coal mines, and other minerals and improvements upon the company's lands, accompanied by maps, together with the report of a survey and exploration of the lands now belonging to the company, made by Walter R. Johnson. Baltimore, Henry E. Huber, 1860. 40 p. And further extracted in: Description of valuable coal, iron ore, lumber, and farm land in Somerset County, Pennsylvania, with geological reports, sections, analysis, &c. by Profs. Lesley, Egleston, Rogers and Johnson and also A. Winters, W. G. Witt, J. Witte, esqs. and others. Philadelphia, E. C. Markley & Son, printers, 1865. 66 p.

62. Report of an examination of the Bear Valley coal district in Dauphin County, Pennsylvania. Philadelphia, Joseph and William Kite, 1841. 36 p.

63. [Some statements respecting foliated galena and fossils from Pennsylvania.] Boston Society of Natural History. *Proceedings* 1: 43–45, 1841.

64. Some observations on the mechanical structure of coal, with evidences of the contemporaneous origin of its various kinds. *Proc. Acad. Nat. Sci. Phila.* 1: 9–12, March/April, 1841; also *J. Acad. Nat. Sci. Phila.* 8: 173–178, 1842.

Reprinted in item 120, *Coal Trade of British America*, 1850, pp. 146–149.

65. Action of light. *Proc. Acad. Nat. Sci. Phila.* 1: 16, May, 1841.

66. Examination and analysis of coal found in the Province of Arauco, coast of Chili, thirty miles south of Bio Bio River. *Proc. Acad. Nat. Sci. Phila.* 1: 21–23, May, 1841; also *J. Acad. Nat. Sci. Phila.* 8: 180–182, 1842.

67. Remarks on the magnetic phenomena attending the congealing of cast-iron. *Proc. Acad. Nat. Sci. Phila.* 1: 33–35, June, 1841.

68. On the relation between the coal of South Wales and that of some Pennsylvania anthracites. *Proc. Acad. Nat. Sci. Phila.* 1: 40–42, June, 1841; also *J. Acad. Nat. Sci. Phila.* 8: 195–198, 1842.

Reprinted in item 120, *Coal Trade of British America*, 1850, pp. 153–154.

69. Remarks on the recent application of anthracite to the smelting of the magnetic iron ores of New Jersey. *Proc. Acad. Nat. Sci. Phila.* 1: 42–43, June, 1841.

70. [On some specimens of *crystalloid* anthracite and bituminous coal.] *Proc. Acad. Nat. Sci. Phila.* 1: 73–75, Aug., 1841.

Reprinted in item 120, *Coal Trade of British America*, 1850, pp. 149–151.

71. Remarks on the samples of anthracite from Rhode Island. *Proc. Acad. Nat. Sci. Phila.* 1: 118–119, Nov., 1841.
Reprinted in item 120, *Coal Trade of British America*, 1850, pp. 151–153.

72. [An apparatus to illustrate the phenomena of the rotation of fluids.] *Proc. Acad. Nat. Sci. Phila.* 1: 121–122, Nov., 1841.

73. [An apparatus, illustrative of the laws of chemical combination and of the combining volumes of gaseous substances.] *Proc. Acad. Nat. Sci. Phila.* 1: 127–130, Nov., 1841.

74. Remarks on the Bear Valley coal district in Dauphin County, Pennsylvania. JOURNAL FRANKLIN INSTITUTE 32: 318–327, Nov., 1841.

75. Observations on the spontaneous combustion of bituminous coal. *Proc. Acad. Nat. Sci. Phila.* 1: 140, Jan., 1842.

76. Results of some experiments on the magnetism of bars of wrought iron broken by tensile force. *Proc. Acad. Nat. Sci. Phila.* 1: 144–145, Jan., 1842.

77. On the practical determination of the heating power of fuel. National Institution for the Promotion of Science. *Bulletin of Proceedings* no. 2: 165–168, Feb., 1842.
Reprinted in item 120, *Coal Trade of British America*, 1850, pp. 137–141.

78. [Analysis of a number of anthracite and bituminous coals of Europe and the U. S.] *Proc. Acad. Nat. Sci. Phila.* 1: 156, March, 1842.
Reprinted in item 120, *Coal Trade of British America*, 1850, pp. 154–155.

79. Determination of copper. *Proc. Acad. Nat. Sci. Phila.* 1: 187, July, 1842.

80. A method of applying the galvanometer to the purpose of testing the relative destructibility of different samples of sheathing copper when exposed to the action of salt water. *Proc. Acad. Nat. Sci. Phila.* 1: 210–211, Sept., 1842.

81. [Trials to determine the volatile and earthy ingredients of the so-called natural coke from Virginia.] *Proc. Acad. Nat. Sci. Phila.* 1: 223–224, Dec., 1842.
Reprinted in item 120, *Coal Trade of British America*, 1850, pp. 155–156.

82. Report of the Commissioners appointed by the Secretary of the Navy to examine the several plans of floating docks submitted to the Department. n.p., 1842. 20 p.
Report signed: Beverly Kennon, Samuel Humphreys, Walter R. Johnson. Another edition, with variations and not as complete, is the following: Report of the Commissioners appointed by the Secretary of the Navy to institute a comparison between the floating drydocks. Washington, 1843. 15 p.

83. Report of the Secretary of the Navy, in compliance with a resolution of the Senate, in relation to the invention of Thomas S. Easton, for preventing explosions of steam boilers. n.p. Thomas Allen, print., 1842. 38 p. (27th Cong. 2d Sess. Senate 336.)
The inclosed report is signed Walter R. Johnson.

84. Report from the Secretary of the Navy communicating, in compliance with a resolution of the Senate, a report on the second invention of Thomas S. Easton, for preventing explosions of steam-boilers. n.p., Thomas Allen, print., 1842. 10 p. (27th Cong. 2d Sess. [Senate 439].)
The inclosed report is signed Walter R. Johnson.
Also printed in: Explosion of steam boilers. Memorial of Thomas S. Easton in relation to explosions of steam boilers. n.p., 1843. 11 p. (27th Cong. 3d Sess. House of Representatives. Doc. no. 59.)
Items 83 and 84 were apparently issued together in the following: Reports to the Secretary of the Navy, in compliance with a resolution of the Senate, in relation to the inventions of T. S. Easton, for preventing explosions of steam boilers. New York, 1842. Not seen but cited in Institution of Civil Engineers. *Catalogue of the library.* London, 1895. Vol. H-PA, p. 162.

85. Report to the Honorable Secretary of the Navy, of researches on the character and tests of sheathing copper, by Walter R. Johnson. In Report of the Secretary of the Navy, Dec. 1842, in Message from the President of the United States to the two Houses of Congress at the commencement of the third session of the twenty-seventh Congress. Dec. 7, 1842. Washington, 1842, pp. 679–721. (27th Cong. 3d Sess. House doc. no. 2, pp. 679–721.)

86. A system of natural philosophy, designed for the use of schools and academies, on the basis of The Book of Science, by Mr. J. M. Moffat; comprising mechanics, hydrostatics, hydraulics, pneumatics, acoustics, pyronomics, optics, electricity, galvanism, magnetism, with emendations, notes, questions for examination, lists of words for reference, some additional illustrations and an index, by Walter R. Johnson. Eighth edition. Philadelphia, E. C. & J. Biddle, 1842. 473 p.

Originally issued as *Scientific class-book*, Part 1. See item 38. This appears with various dates. 1845 and 1847 have also been noted.

87. An elementary treatise on chemistry, together with treatises on metallurgy, mineralogy, chrystallography, geology, oryctology and meteorology. Designed for the use of schools and academies, on the basis of The Book of Science, by Mr. J. M. Moffat. With additions, emendations, notes, references, questions for examinations, lists of works relating to the several subjects, some additional illustrations, and a copious index. Eighth edition. Philadelphia, E. C. & J. Biddle, 1846. 478 p.

Originally issued as *Scientific class-book*, Part 2. See item 38.

88. [On an apparatus for testing tenacity of sheathing copper.] *Proc. Acad. Nat. Sci. Phila.* 1: 245–246, Feb., 1843.

89. Observations in relation to the properties and habitudes of different varieties of coal. *Proc. Acad. Nat. Sci. Phila.* 2: 8–10, Feb., 1844.

90. [On different varieties of plumbago.] *Proc. Acad. Nat. Sci. Phila.* 2: 74–75, July, 1844.

91. [On some specimens of rocks from the White Mountains.] *Proc. Acad. Nat. Sci. Phila.* 2: 89–90, Oct., 1844.

92. [On the spontaneous combustion of bituminous coal.] *Proc. Acad. Nat. Sci. Phila.* 2: 163–164, Dec., 1844.

93. A memoir on the scientific character and researches of the late James Smithson, esq. F.R.S. . . . Read before the National Institute, Washington, April 6, 1844. Philadelphia, Barrett and Jones, 1844. 25 p.

Another edition has 36 p. of which pp. 26–36 contain the text of item 47, "Establishment of an institution for experiments in physical sciences." Reprinted in William J. Rhees, *The Scientific Writings of James Smithson*. Washington, 1879, pp. 123–141. (Smithsonian miscellaneous collections 327.)

94. A report to the Navy Department of the United States, on American coals applicable to steam navigation, and to other purposes. Washington: Gales and Seaton. n.d. xii, 607 p.

Another edition. This has "28th Congress, 1st Session" [Senate 386] at head of title and date 1844 at bottom.

Another edition. —Washington, Blair & Rives, 1844. iv, 607 p. (28th Cong. 1st Sess. House doc. no. 276.)

Pp. v-xii of the first two editions listed comprise "Preliminary report of experiments on the evaporative power and other properties of American coals. 1843." This was published earlier as "Report of Professor W. R. Johnson" in "Report of the Secretary of the Navy, 1843," pp. 679–686.

Reviewed with extracts in **Am. J. Sci. Arts* 49: 310–336, July–Sept., 1845. Noticed with brief extracts under title "Evaporative power of different kinds of coal and their relative values" in *Civil Engineer and Architect's Journal* 8: 242–245, Aug., 1845, and from this translated into German in *Dingler's Polytechnisches Journal* 98: 132–140, 1845, and in *Polytechnisches Centralblatt*, n.f. 7: 365–375, 1846.

95. Letter from Professor W. R. Johnson on the subject of further experiments on iron, copper, and coal. In Report of the Secretary of the Navy, Nov. 25, 1844, in Message from the President of the United States to the two Houses of Congress, at the commencement of the second session of the twenty-eighth Congress. Dec. 3, 1844. Washington, Gales & Seaton, 1844, pp. 639–644. (28th Cong. 2d Sess. Senate Doc. 1, pp. 639–644.)

Same (28th Cong. 2d Sess. House of Representatives. Ex. Doc. 2, pp. 637–641.)

96. Report of the Secretary of the Navy communicating the report of a board of examiners to make experimental trials of inventions and plans for preventing explosions of steam boilers. n.p., 1844. 52 p. (28th Cong. 1st Sess. Senate 405.)

Report of board signed Thomas P. Jones, Walter R. Johnson.

97. [On the calorific powers of combustible bodies.] *Proc. Acad. Nat. Sci. Phila.* 2: 202–206, Feb., 1845.

Reprinted in *Am. J. Sci. Arts* 49: 166–171, April–June, 1845.

98. [On the tendency to cohesion of metallic surfaces when under magnetic influences.] *Proc. Acad. Nat. Sci. Phila.* 2: 209–210, March, 1845.

99. Observations on the relative evaporation from land and water surfaces. *Proc. Acad. Nat. Sci. Phila.* 2: 295–298, Dec., 1845.

100. Examination and analyses of samples of the alluvial soil of the Nile, from Korosco, in Nubia. *Proc. Acad. Nat. Sci. Phila.* 2: 318–324, Dec., 1845.

Reprinted in following: On the Nile alluvium of Nubia. Extract of a letter from Dr. Richard Lepsius. . . . To Dr. S. G. Morton, relative to the language of the Bishareens of Nubia, and the alluvial deposits of the Nile. With an analysis of those deposits by Prof. Walter R. Johnson. [Philadelphia, 1845.] 8 p.

101. Abstract of Professor Johnson's report to the Secretary of the Navy of the United States, respecting Forest Improvement coal. New York, George F. Nesbitt, 1845. 1 l., 8 p.

The leaf is tipped in and dated 1846.

102. Report of the commissioners appointed by authority of the City Council to examine the sources from which a supply of pure water may be obtained for the City of Boston. Boston, J. H. Eastburn, 1845. 128, xxxii p.

Report signed John B. Jervis, Walter R. Johnson.

103. Effect of heat on the tenacity of iron. *Am. J. Sci. Arts* 2d ser. 1: 299–300, March, 1846.

From the [Report on the strength of materials for steam-boilers] 1837, item 45. As a separate has title: "Thermotension, or the effect of heat on the tenacity of iron."

104. A report on the coal-mines and coal-trade of Nova Scotia and New Brunswick. n.p., Nov. 14th, 1846. 133 p.

Manuscript copy in University of Pennsylvania Library. "Copied from original manuscript report by Florence V. Strecker. August 13th, 1873." Printed in item 120, *Coal Trade of British America*, 1850, pp. 9–54, with addition of 2 sections.

105. [Salts of ammonia in the dust of anthracite furnaces.] *Proc. Acad. Nat. Sci. Phila.* 3: 191, March, 1847.

Reprinted in item 120, *Coal Trade of British America*, 1850, pp. 145–146.

106. Use of anthracite coal in locomotives. JOURNAL FRANKLIN INSTITUTE 44: 110–116, Aug., 1847.

Reprinted in *Mechanics' Magazine* 47: 411–414, Oct. 23, 1847.

107. On a boiler sediment. *Proc. Acad. Nat. Sci. Phila.* 3:318, Nov., 1847.

108. A section of the coal seams and accompanying measures of the Hazelton coal basin in Luzerne County, Penn., as an illustration of the construction and coloring of geological sections. *Am. J. Sci. Arts* 2d ser. 5: 111–113, Jan., 1848.

Abstract of a paper presented at meeting of Association of American Geologists and Naturalists. Boston, Sept., 1847.

109. On some recent improvements in recording magnetic telegraphs. *Am. Assn. Advancement Sci. Proc.* 1st meeting, 93–94, 1848.

110. Principles of the mechanics of machinery and engineering by Julius Weisbach. First American edition edited by Walter R. Johnson. Philadelphia, Lea and Blanchard, 1848–49. 2 v.

Volume 1 reviewed in JOURNAL FRANKLIN INSTITUTE 45: 389, May, 1848.

111. Chemical technology; or, chemistry applied to the arts and to manufactures, by Dr. F. Knapp, translated and edited with numerous notes and additions by Dr. Edmund Ronalds and Dr. Thomas Richardson. First American edition, with notes and additions by Professor Walter R. Johnson. Philadelphia, Lea and Blanchard, 1848–49. 2 v.

Reviewed in JOURNAL FRANKLIN INSTITUTE 45: 449–451, June, 1848.

112. A lecture on the mechanical industry and the inventive genius of America. Delivered before the Maryland Institute for Promoting the Mechanic Arts, Baltimore, January 16, 1849. Baltimore, Sands & Mills, n.d. 23 p.

Extract published under the title: "Mechanical industry and inventive genius of America." *American Quarterly Register and Magazine* 2: 404–407, June, 1849.

Extract published under the title: "Fruits of American genius—United States letters patent." *Niles' National Register* 75: 380–381, June 13, 1849.

113. Russian coal. *Niles' National Register* 75: 389, June 20, 1849.

114. On some experimental determinations of the economic values of British and American coals. *Am. Assn. Advancement Sci. Proc.* 2d meeting, 221–231, 1849.

As separate with title: "Experimental determinations of the economic values of British and American coals." 8 p.

115. [Observations on the apparent transfer of motion of a moving train or car to an observer in a stationary car.] *Am. Assn. Advancement Sci. Proc.* 2d meeting, 376–377, 1849.

116. Some observations on the gold formations of Maryland, Virginia and North Carolina. *Am. Assn. Advancement Sci. Proc.* 4th meeting, 20–21, Aug., 1850.

117. Scientific interest of the proposed industrial exhibition at London, in 1851. *Am. Assn. Advancement Sci. Proc.* 4th meeting, 242, Aug., 1850.

118. On the coal formation of central North Carolina. *Am. Assn. Advancement Sci. Proc.* 4th meeting, 274–276, Aug., 1850.

119. Report on Prof. Page's electro-magnetic engine. *Am. J. Sci. Arts* 2d ser. 10: 473–476, Nov., 1850.

120. The coal trade of British America, with researches on the characters and practical values of American and foreign coals. Washington, Taylor & Maury; Philadelphia, A. Hart, 1850. 179 p.

Includes items 64, 68, 70, 71, 77, 78, 81, 104, 105.

121. Report of an examination of the coal and iron ore lands known as the Wilson survey, lying on the south side of the Great Kanawha River, in the counties of Kanawha and Fayette, state of Virginia. [Washington, Buell & Blanchard, 1850] 8 p.

122. * Comparison of experiments on American and foreign building stones to determine their relative strength and durability. *Am. J. Sci. Arts* 2d ser. 11: 1–17, Jan., 1851.

123. On some theoretical and practical methods of determining the calorific efficiencies of coals. *British Association for the Advancement of Science. Report.* 21st meeting (pt. 2), 47–51, 1851.

124. Report on the coal lands of the Deep River Mining and Transportation Company, in Chatham and Moore counties, North Carolina. With analyses of the minerals. By Walter R. Johnson. Albany, Weed, Parsons and company, 1851. 39 p.

Reprinted, with omissions, under title: "The coal lands of the Deep River Company in North Carolina; with analyses of the minerals." *Mining Magazine* 1: 352–365, Oct., 1853.

125. Report of Professor Walter R. Johnson on the building stone used in constructing the foundations of the extension of the United States Capitol made at the request of a select committee of the House of Representatives . . . March 11, 1852. In Report of the Secretary of the Interior, 1852 in Message from the President of the United States to the two Houses of Congress at the commencement of the second session of the thirty-second Congress. Washington, 1852, pp. 594–606. (32nd Cong. 2d Sess. Senate. Ex. Doc. 1.)

Same (32nd Cong. 2d. Sess. House of Representatives. Ex. Doc. 1.)

Pocket Instrument Indicates Relative Humidity.—A precision pocket instrument, shorter than a pencil, that shows relative humidity by the accurate wet-and-dry bulb method has been developed by the Weston Electrical Instrument Corp. Known as the "TAG" Pocket Humidicator Model 8704, this compact new instrument can be used either in the hand or mounted on a wall. A simple built-in slide rule converts the wet-bulb and dry-bulb readings directly into relative humidity, eliminating charts or tables.

This new "TAG" device consists of two etched scale glass thermometers, individually hand-calibrated for permanently accurate readings. These are secured in a sturdy white plastic case by integral resilient mountings. The psychometric slide rule forms the cover of this case. For use, a few drops of water are applied to the thermometer wick, and the instrument is fanned or swung for several moments. Setting the wet-bulb reading opposite the dry-bulb reading on the slide rule then indicates relative humidity.

The "TAG" Pocket Humidicator has a wide field of application in heating, air conditioning, ventilating, refrigeration, industrial and chemical processing where the per cent of atmospheric humidity is a factor.

Cell Regulator.—A biological cell regulator capable of speeding up the life processes of cells as much as 100 per cent has been discovered by University of Wisconsin biochemists Drs. Henry A. Lardy and D. Ghosh. The substance is found in all mammalian sperm cells, is manufactured by the cells themselves, and is apparently utilized as needed by the cells to regulate life processes. In experiments, Professor Lardy showed that excessive amounts of the regulator would stimulate the fermentation of yeast cells as much as two times over normal. An interesting characteristic of the regulator is that it stimulates cell respiration and the breaking down of glycogen by cells, while at the same time preventing energy utilization. The American Cancer Society is supporting Professor Lardy's work to find how and why the regulator influences cellular growth and division.

The regulator has been purified from bull semen, one of the best sources. Automatic liberation of the regulator in the material used in artificial insemination of cattle causes senescence—old age—of the sperm cells. Methods to prevent liberation of the regulator would enable artificial insemination operators to preserve semen for much longer periods of time than is now possible, Professor Lardy points out.

The Wisconsin scientists are working to find a substance to inhibit the regulator and extend the time span of bull semen potency.

Professors Lardy and Paul H. Phillips, also of the Wisconsin biochemistry department, developed the artificial medium for preservation of bull semen now used by inseminators throughout the world.

Existence of a sheet of electric currents 66 miles up, which apparently flows like ocean tides, has been confirmed in high-altitude rocket research by scientists at the Naval Ordnance Laboratory, White Oak, Md. This layer, long an accepted theory, is believed to explain variations in radio wave strengths. Motion of the charges may be responsible for continuous changes in the magnetic field of the earth.

DIFFERENTIAL ANALYZER SOLUTION FOR THE STRESSES IN A ROTATING BELL-SHAPED SHELL

BY

J. L. MERIAM [1]

ABSTRACT

In an earlier paper (1)[2] by the author the theory of axially symmetrical shells was extended to include the body force of rotation about the axis of generation, and the results were applied to the conical shell. In the present paper this theory is used to obtain the solution for a bell-shaped rotating shell whose middle surface is generated by revolving a circular arc about a tangent line. The defining relation is an ordinary linear differential equation of fourth order with variable coefficients. A differential analyzer solution was used and this involved the problem of handling boundary conditions which were divided between both extremities of the range of the independent variable. Comparisons of the results with those of the conical shell and with the membrane theory are made.

NOMENCLATURE

The following nomenclature is used in the paper:

- r_1 = constant radius of meridian curvature,
- h = shell thickness,
- β = angle between normal to axis of rotation and normal to shell meridian,
- θ = angle measured about axis of rotation in plane of rotation,
- ρ = mass per unit area of shell,
- ω = angular velocity of shell,
- Q = shear force per unit length of shell section,
- N_β = membrane force along meridian per unit length of shell section,
- N_θ = membrane force tangent to a parallel circle per unit length of shell section ("hoop stress"),
- M_β = bending moment in plane of meridian per unit length of shell section,
- M_θ = bending moment in plane perpendicular to meridian per unit length of shell section,
- R = body force of rotation per unit area of shell,
- v = displacement component along meridian,
- w = displacement component normal to meridian,
- ζ = radial displacement (normal to axis of rotation),
- e_β = strain in direction of meridian,
- e_θ = strain in direction tangent to a parallel circle,
- χ_β = change in curvature in plane of meridian,
- χ_θ = change in curvature in plane perpendicular to meridian,

[1] Assistant Professor of Engineering Design, University of California, Berkeley, Calif.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

ν = Poisson's ratio,
E = modulus of elasticity, and
D = flexural rigidity.

Dimensionless quantities:

$Q' = Q/\rho\omega^2 r_1^2$
$N_\beta' = N_\beta/\rho\omega^2 r_1^2$
$N_\theta' = N_\theta/\rho\omega^2 r_1^2$
$M_\beta' = M_\beta/\rho\omega^2 r_1^3$
$M_\theta' = M_\theta/\rho\omega^2 r_1^3$
$\zeta' = \zeta/(r_1/Eh)$
$h' = h/r_1$

INTRODUCTION

Solutions for rotors which are symmetrical with respect to a plane of rotation and whose axial dimensions are small compared with the radial

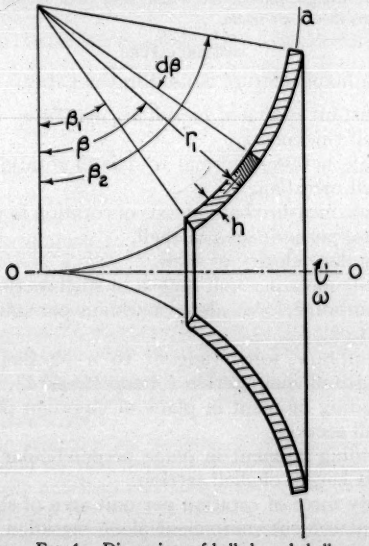


FIG. 1. Dimensions of bell-shaped shell.

dimensions are well known. The solution for the rotating conical shell (1) was a departure from this condition of symmetry. The analysis of the bell-shaped shell reported in this paper represents a still further departure and approximates the curved profiles of many of the high speed pump impellers. Analyses of ring- or torus-shaped shells under static loading are available in the papers of Wissler (2) and Stange (3). The shell of the present investigation differs in that it is subject to a

body force due to rotation and that its surface is defined by a portion of a torus with zero hole.

The defining equations for the bell-shaped shell can be obtained from the relations of the earlier paper (1) by appropriate changes in variables and symbols. These substitutions are listed in the Appendix. In the present article the essential equations are cited to maintain the completeness of the presentation and to provide necessary reference.

THEORY

The middle surface of the shell is shown in Fig. 1 and is generated by revolving the circular meridian arc *oa* of radius r_1 about the tangent

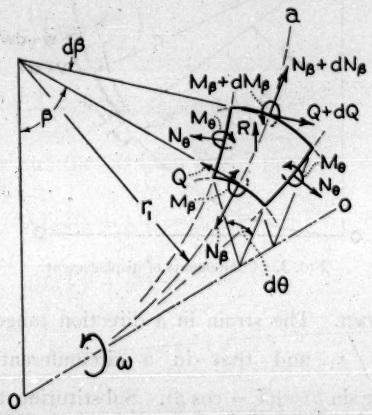


FIG. 2. Shell element with forces and moments. (Shell thickness not shown.)

line *oo*. The angles β_1 and β_2 define the inner and outer boundaries, respectively, and the thickness h is assumed small compared with r_1 but large enough to support appreciable bending stresses. Rotation about the axis *oo* with a constant angular velocity provides the centrifugal loading. An element of the shell with the forces and moments acting on it is shown in Fig. 2. Equating to zero the forces in a direction tangent to the arc *oa* gives

$$\frac{d}{d\beta}\left(N_\beta[1-\cos\beta]\right) - N_\theta \sin\beta + Q(1-\cos\beta) + \rho\omega^2 r_1^3 \sin\beta(1-\cos\beta)^2 = 0. \quad (1)$$

Likewise, equilibrium of forces in the direction of the radius r_1 gives

$$\frac{d}{d\beta}\left(Q[1-\cos\beta]\right) + N_\theta \cos\beta - N_\beta(1-\cos\beta) - \rho\omega^2 r_1^3 \cos\beta(1-\cos\beta)^2 = 0. \quad (2)$$

A third equation is obtained by equating to zero the moments about the upper edge and

$$\frac{d}{d\beta}\left(M_\beta[1-\cos\beta]\right) - M_\theta \sin\beta - Qr_1(1-\cos\beta) = 0. \tag{3}$$

Equations 1, 2, and 3 completely specify the equilibrium of the shell element.

In Fig. 3 the positive directions of the two displacements

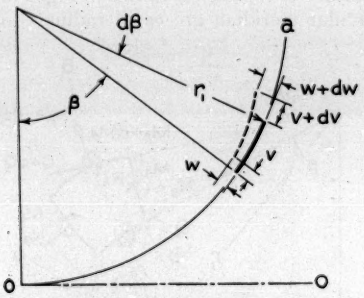


FIG. 3. Components of displacement.

v and w are shown. The strain in a direction tangent to the arc is $e_\beta = \left(\frac{dv}{d\beta} - w\right)\Big/ r_1$ and that in a circumferential direction is $e_\theta = (w\cos\beta + v\sin\beta)/r_1(1-\cos\beta)$. Substituting these expressions into the Hooke's Law equations gives

$$\left.\begin{aligned} N_\beta &= \frac{Eh}{1-\nu^2}\left[\frac{1}{r_1}\left(\frac{dv}{d\beta} - w\right) + \frac{\nu(w\cos\beta + v\sin\beta)}{r_1(1-\cos\beta)}\right] \\ N_\theta &= \frac{Eh}{1-\nu^2}\left[\frac{w\cos\beta + v\sin\beta}{r_1(1-\cos\beta)} + \frac{\nu}{r_1}\left(\frac{dv}{d\beta} - w\right)\right]. \end{aligned}\right\} \tag{4}$$

The moments are related to the changes in curvature by the equations

$$\left.\begin{aligned} M_\beta &= -D(\chi_\beta + \nu\chi_\theta) \\ M_\theta &= -D(\chi_\theta + \nu\chi_\beta), \end{aligned}\right\} \tag{5}$$

where

$$\left.\begin{aligned} \chi_\beta &= -\frac{1}{r_1^2}\frac{d}{d\beta}\left(v + \frac{dw}{d\beta}\right) \\ \chi_\theta &= -\frac{1}{r_1^2}\operatorname{ctn}\beta\,(\sec\beta + 1)\left(v + \frac{dw}{d\beta}\right). \end{aligned}\right\} \tag{6}$$

Equations 1 through 6 completely define the problem. Their combination is somewhat lengthy (1) but results in the two equations

$$\left.\begin{aligned} L(U) - \frac{\nu}{r_1} U &= VEh + T \\ L(V) + \frac{\nu}{r_1} V &= -\frac{U}{D} \end{aligned}\right\} \tag{7}$$

where

$$L(\ ) = \frac{\sec\beta - 1}{r_1}\frac{d^2(\ )}{d\beta^2} + \frac{\tan\beta}{r_1}\frac{d(\ )}{d\beta} - \frac{\sec\beta + 1}{r_1}(\ )$$

$$U = r_1(\sec\beta - 1)Q$$

$$V = -\frac{1}{r_1}\left(v + \frac{dw}{d\beta}\right)$$

and

$$T = \frac{3 + \nu}{2}\rho\omega^2 r_1^2 \sin 2\beta(\sec\beta - 1)^2.$$

Eliminating first V and then U from Eqs. 7 gives

$$\left.\begin{aligned} LL(U) + \mu^4 U &= L(T) + \frac{\nu}{r_1} T \\ LL(V) + \mu^4 V &= -\frac{T}{D} \end{aligned}\right\} \tag{8}$$

where

$$\mu^4 = \frac{Eh}{D} - \frac{\nu^2}{r_1^2}.$$

Either of Eqs. 8 may be used to solve the problem. The homogeneous or reduced equation is the same for each but the particular integral is different. Since the first of the two involves the component of force Q it will be used in favor of the second which is expressed in terms of the rotation V of a shell element.

Carrying out the operations indicated in the first of Eqs. 8 and substituting the dimensionless shear $Q' = Q/\rho\omega^2 r_1^2$ results in the defining differential equation

$$\frac{d^4Q'}{d\beta^4} + j_3\frac{d^3Q'}{d\beta^3} + j_2\frac{d^2Q'}{d\beta^2} + j_1\frac{dQ'}{d\beta} + j_0Q' = j \tag{9}$$

where

$$j_3 = 2\ \mathrm{ctn}\ \beta(\sec\beta + 1)(3\sec\beta + 1)$$

$$j_2 = (\sec\beta + 1)^2\{\mathrm{ctn}^2\beta(13\sec^2\beta - 1) + \csc^4\beta(7 - 5\cos\beta - 2\cos^2\beta)\}$$

$$j_1 = (\sec\beta + 1)^2\{\csc^3\beta(28\sec\beta - 8 - 5\cos\beta) + \csc\beta(12\sec\beta + 4 + 2\cos\beta)\}$$

$$j_0 = (\sec\beta + 1)^3\{\csc^6\beta(6\sec\beta - 9 + \cos\beta + 2\cos^2\beta) + \csc^4\beta(3[10\sec\beta - 11] + 2\sin^2\beta[2\sec\beta - 1] + \cos\beta + 2\cos^2\beta) - \operatorname{ctn}^2\beta(\sec\beta + 1)\} + \frac{r_1^2\mu^4}{(\sec\beta - 1)^2}$$

$$j = (3 + \nu)(1 + \cos\beta)^2\operatorname{ctn}^3\beta\{\sec^3\beta(2\sec\beta + 1) + (2 + \nu)\sec\beta - (5 + \nu)\}.$$

When there are no membrane or shear forces on the boundary $\beta = \beta_2$, equilibrium of the shell requires that

$$N_\beta' = -Q'\tan\beta. \tag{10}$$

From either Eq. 1 or 2 and Eq. 10 is obtained

$$N_\theta' = (1 - \cos\beta)^2 - (\tan\beta\sec\beta)Q' - (\sec\beta - 1)\frac{dQ'}{d\beta}. \tag{11}$$

The bending moments are derived from Eqs. 5 and 6 using the substitution $V = -\left(v + \frac{dw}{d\beta}\right)\Big/ r_1$ and the first of Eq. 7 relating V and U.

Expressing U in terms of Q and reducing to dimensionless form there results

$$M_\beta' = -p\left[k_3\frac{d^3Q'}{d\beta^3} + k_2\frac{d^2Q'}{d\beta^2} + k_1\frac{dQ'}{d\beta} + k_0Q' - k\right] \tag{12}$$

where

$$p = \frac{h'^2}{12(1 - \nu^2)}$$

$$k_3 = (\sec\beta - 1)^2$$

$$k_2 = \tan\beta\,(\sec\beta - 1)\,(4\sec\beta + 1 + \nu)$$

$$k_1 = (\sec\beta - 1)\{(2\sec\beta + 1)(3\sec^2\beta + [1 + \nu]\sec\beta - [1 - \nu] + \sec\beta\,(2\sec^2\beta - 1) - \nu\}$$

$$k_0 = \tan\beta\sec\beta\,(\sec\beta - 1)(6\sec^2\beta + 2\sec\beta - 1) + \tan\beta(\sec\beta + \nu)(\sec\beta[2\sec^2\beta - 1] + \sec^2\beta - [1 + \nu])$$

$$k = (3 + \nu)(\sec\beta - 1)^2(\sec\beta + 1)\cos^2\beta\{2 + \nu + \csc^2\beta\,(\sec\beta - 1)\}.$$

Also

$$M_\theta' = -p\left[m_3\frac{d^3Q'}{d\beta^3} + m_2\frac{d^2Q'}{d\beta^2} + m_1\frac{dQ'}{d\beta} + m_0Q' - m\right] \tag{13}$$

where

$$m_3 = \nu(\sec\beta - 1)^2$$

$$m_2 = \tan\beta(\sec\beta - 1)(4\nu\sec\beta + 1 + \nu)$$

$$m_1 = (\sec\beta - 1)\{(2\sec\beta + 1)(\sec\beta + 1) + \nu(8\sec^3\beta + 5\sec^2\beta - 2\sec\beta - 1 - \nu)\}$$

$$m_0 = \tan\beta\{2\sec^3\beta + \sec^2\beta - \sec\beta - 1 - \nu + \nu\sec\beta(8\sec^3\beta - 3\sec^2\beta - 4\sec\beta - \nu)\}$$

$$m = (3 + \nu)(\sec\beta - 1)\{\nu(\sec\beta - \cos 2\beta) + \sin^2\beta\}.$$

The radial deflection of any element is $\zeta = e_\theta r_1(1 - \cos\beta)$. Substituting the Hooke's Law relation gives

$$\zeta = \frac{r_1(1 - \cos\beta)}{Eh}(N_\theta - \nu N_\beta).$$

Changing to dimensionless form, expressing N_θ' and N_β' in terms of Q', and using Eqs. 10 and 11 give

$$\zeta' = \frac{\zeta}{r_1/Eh} = \cos\beta(\sec\beta - 1)^2\left[-\frac{dQ'}{d\beta} - f_0Q' + f\right] \qquad (14)$$

where

$f_0 = \operatorname{ctn}\beta(\sec\beta + 1)(\sec\beta - 1)$
$f = \cos\beta(1 - \cos\beta)$.

The rotation V of an element is needed in the process of deriving the expressions for M_β' and M_θ'. From the first of Eqs. 7 this becomes

$$V = \frac{\rho\omega^2 r_1^2}{Eh}(\sec\beta - 1)^2\left[\frac{d^2Q'}{d\beta^2} + e_1\frac{dQ'}{d\beta} + e_0Q' - e\right] \qquad (15)$$

where

$e_1 = \operatorname{ctn}\beta(\sec\beta + 1)(2\sec\beta + 1)$
$e_0 = (\sec\beta + 1)[1 + \nu + \sec\beta + \csc^2\beta(\sec\beta - \nu)]$
$e = \frac{3 + \nu}{2}\sin 2\beta.$

In Eqs. 10 through 15 the membrane forces, the moments, the radial deflection, and the rotation are all expressed in terms of Q' and its derivatives.

SOLUTION OF THE DIFFERENTIAL EQUATION

The homogeneous part of Eq. 8 can be factored to

$$L(U) \pm i\mu^2 U = 0.$$

Thus, solution of the homogeneous equation is reduced to the solution of these two second order equations which, although simpler than Eq. 9, are still awkward for series expression when expanded. Wissler (2) and Stange (3) developed solutions of their similar second order linear equations but the resulting functions necessitate extensive calculation. The particular integral involved in the present shell also adds considerable complexity to the form of a rigorous mathematical solution.

The availability of the differential analyzer at the University of California at Los Angeles suggested the investigation of a machine solution for this type of problem. The resulting advantages and disadvantages of this method will be shown subsequently. In setting up the analyzer it was found that the complete fourth order form, Eq. 9, was more convenient for use than the factored form by reason of the conservation of certain elements of the available analyzer equipment.

The j's of Eq. 9 were first calculated using $\nu = 0.3$ and plotted against β. These functions were then supplied to the machine on input tables manually operated. Due to the rapid changes in j, j_0, j_1, j_2, it was found advisable to supply these functions to the machine in the form of their logarithms. This greatly reduced the manual errors due to the difficulty in following rapidly changing curves. These logarithms were converted by means of one integrator in each case to the function. The j-plots and the schematic machine diagram are shown in Figs. 4 and 5, respectively.

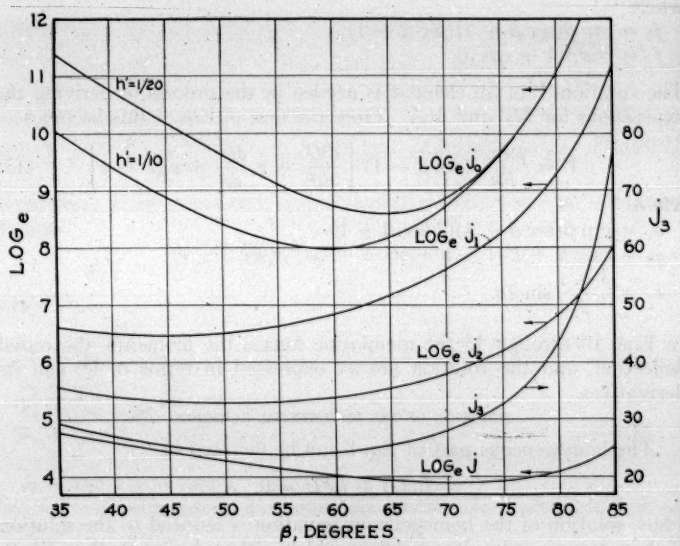


FIG. 4. j-Coefficients of defining differential equation.

Problems of divided boundary conditions present a difficulty when a machine or step-by-step solution is attempted. The difficulty lies in the fact that in order to start such solutions the initial values of the dependent variable and its $n - 1$ derivatives must be assigned at the starting point before a solution may proceed. Thus, in general, it is necessary to adopt a trial and error process selecting various arbitrary values for the initially unknown derivatives until a combination is obtained which will satisfy the boundary conditions at the second point at which boundary conditions are specified. In the case of the fourth order equation of this paper two boundary conditions exist at each of

the two edges $\beta = \beta_1$ and $\beta = \beta_2$ of the shell. Hence if the solution is begun at $\beta = \beta_1$ then two of the derivatives will be unknown. Fortunately a method for handling this problem of two-point boundary values is available for ordinary linear differential equations (4). In the case of a fourth order equation with equally divided boundary conditions the method consists of making three trial solutions with three arbitrary but different combinations of the two unknown derivatives at the starting point. The other derivatives are determined at this point by the two known boundary conditions. Each of the three trial solutions proceeds until the point at which the remaining two conditions are

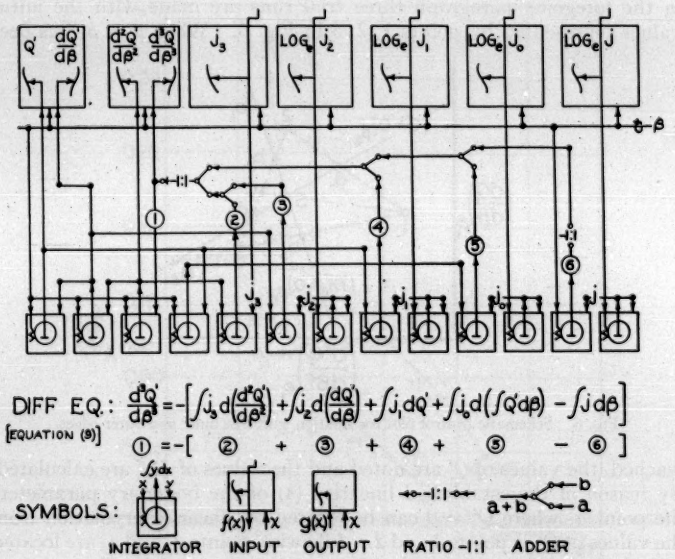


FIG. 5. Schematic machine diagram for solution of differential equation.

known is reached. The two quantities which take on prescribed values at this second boundary are next calculated for each of the three solutions at this point. If these quantities are expressible as linear combinations of the dependent variable and its $n - 1$ derivatives, then by linear interpolation and extrapolation it is possible to determine the proper initial combination of unknown derivatives and also the correct values of the dependent variable and its derivatives for all points within the interval of solution.

To illustrate this method consider the case where the shell rotates with free inner and outer boundaries. Thus the four conditions to be

satisfied are $Q' = M_\beta' = 0$ for both $\beta = \beta_1$ and $\beta = \beta_2$. If the solution begins at β_1, then, in addition to $Q' = 0$, Eq. 12 requires that

$$\left[k_3 \frac{d^3Q'}{d\beta^3} + k_2 \frac{d^2Q'}{d\beta^2} + k_1 \frac{dQ'}{d\beta} - k \right]_{\beta = \beta_1} = 0. \tag{16}$$

The values of any two of the derivatives in Eq. 16 must be assumed and the third calculated so as to satisfy this algebraic equation. If $dQ'/d\beta$ and $d^2Q'/d\beta^2$ are the two arbitrarily selected derivatives, then the possible initial choices may be represented as points when plotted against $(dQ'/d\beta)_{\beta_1}$ and $(d^2Q'/d\beta^2)_{\beta_1}$. In accordance with the method cited in the foregoing paragraph three trial runs are made with the initial values represented by points 1, 2, 3 in Fig. 6. When $\beta = \beta_2$ has been

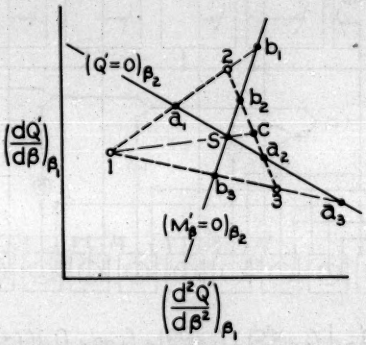


FIG. 6. Schematic control plot for solution with free inner and outer edges.

reached, the values of Q' are noted and the values of M_β' are calculated. By reason of the established linearity (4) of the boundary parameters the point a_1 where $Q' = 0$ can be located by linear interpolation from the values of Q' at points 1 and 2. Likewise, points a_2 and a_3 are located and the parameter line $(Q' = 0)_{\beta_2}$ can be drawn. In like manner, points b_1, b_2, b_3 where $M_\beta' = 0$ are located from the values of M_β' at points 1, 2, 3, and the $(M_\beta' = 0)_{\beta_2}$ parameter line is drawn. The intersection of these two parameters gives the solution point S. Point S determines the proper initial combination of derivatives which would have resulted in the correct solution. Although this solution may then be made using these values it is not necessary to do so. The correct values of Q' and its derivatives may be obtained for all values of β by direct linear interpolation and extrapolation from the corresponding values at points 1, 2, 3 to the point S. This is done, for example, by interpolating to point c from 2 and 3, and then interpolating to S from 1 and c.

In the case of a shell with clamped inner edge and free outer edge Eqs. 14 and 15 require that

$$\left.\begin{aligned} &\frac{dQ'}{d\beta} + f_0 Q' - f = 0 \\ \text{and} \quad &\frac{d^2Q'}{d\beta^2} + e_1 \frac{dQ'}{d\beta} + e_0 Q' - e = 0 \end{aligned}\right\} \tag{17}$$

for $\beta = \beta_1$. Again it is necessary to assume two initial values such as

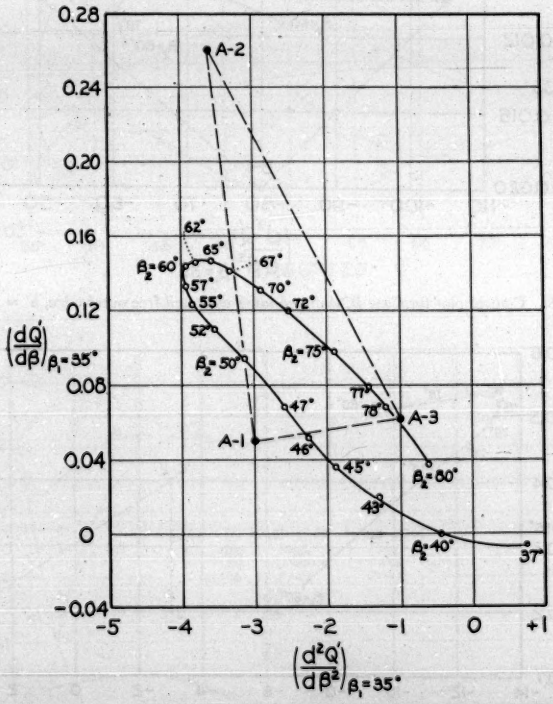


FIG. 7. Control plot for Case A, free inner and outer edges, $h' = 1/20$.

Q' and $d^3Q'/d\beta^3$. The corresponding initial values of the first and second derivatives are obtained from Eq. 17 expressed at β_1. If the outer edge is free, then the relations discussed in the preceeding paragraph hold at β_2. The control points are plotted against the initial values of Q' and $d^3Q'/d\beta^3$.

In the procedure described it is necessary that the distances between trial points be of the same order of magnitude as the distance from the solution point to any of the trial points. Although the error due to extrapolation of non-linear quantities is absent, still small errors in the

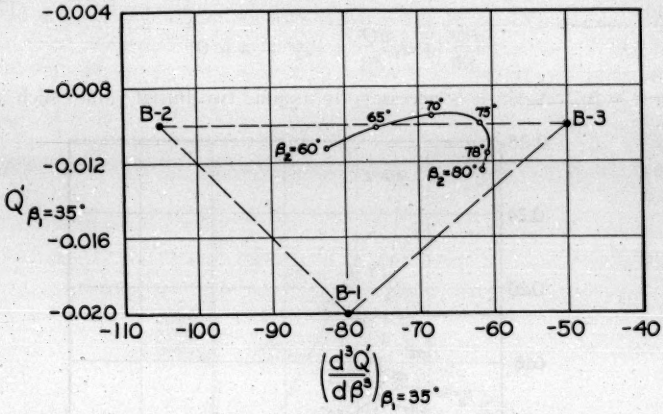


FIG. 8. Control plot for Case *B*, clamped inner edge and free outer edge, $h' = 1/20$.

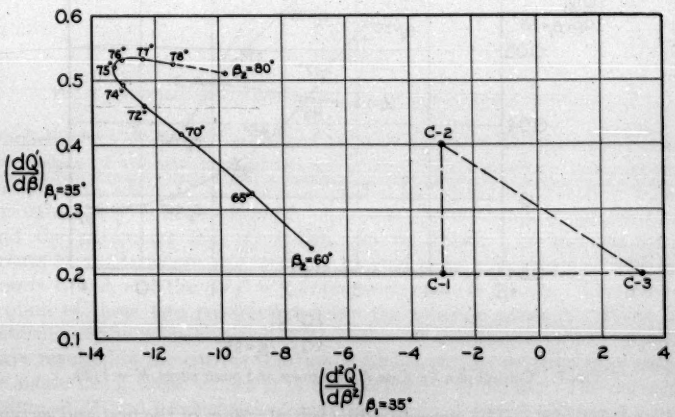


FIG. 9. Control plot for Case *C*, free inner and outer edges, $h' = 1/10$.

values of the trial solutions will limit the extent of accurate extrapolation. Thus a preliminary estimate of the region of solution is very helpful and will alleviate the necessity for repeating the trial process

with three additional points closer to the solution point. In the case of the present problem a sufficiently close estimate of the region of solution was made with the aid of the results for the rotating conical shell (1).

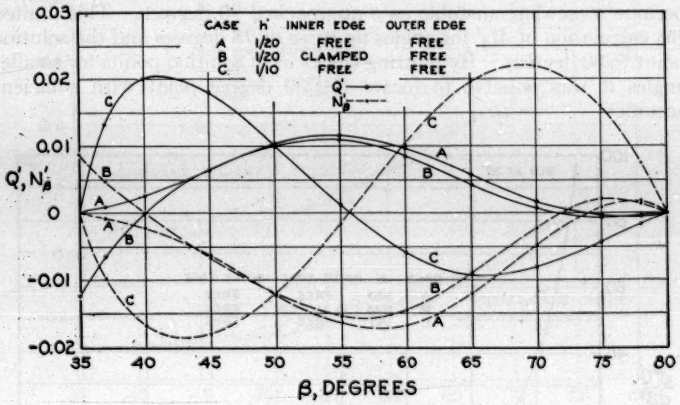


FIG. 10. Variation of shear Q' and membrane force N_β'.

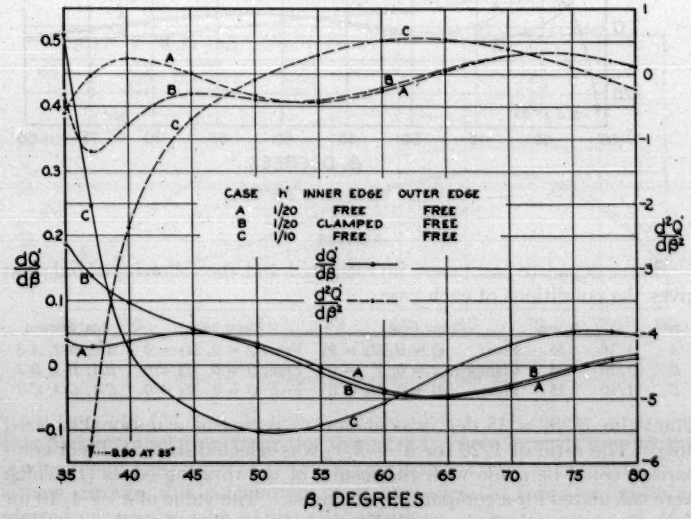


FIG. 11. Variation of $dQ'/d\beta$ and $d^2Q'/d\beta^2$.

In addition to obtaining a solution for a given value of β_2 the data of three runs enable solutions to be made for various values of β_2. Thus a family of solutions may be obtained and indicated by the locus of solution points S on the control plot of Fig. 6.

Due to the rapidly increasing j-coefficients in Eq. 9 the machine became somewhat unstable as β approached 90 degrees. This limited the calculation of M_β' for angles up to 75 or 78 degrees and the solution point to 80 degrees. By plotting the locus of solution points for smaller angles it was possible to locate the 80 degree point with sufficient accuracy.

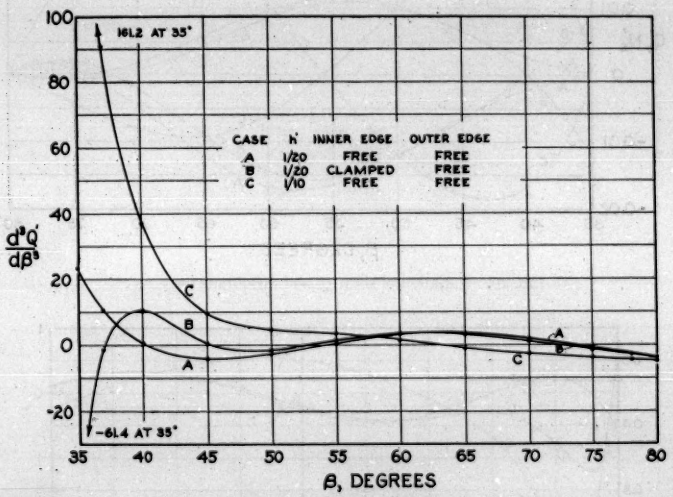


FIG. 12. Variation of $d^3Q'/d\beta^3$.

RESULTS

Three separate cases were investigated and the following tabulation gives the conditions of each case.

Case	h'	β_1, deg.	Inner Edge	Outer Edge	Trial Solution
A	1/20	35	Free; $Q = 0, M_\beta = 0$	Free; $Q = 0, M_\beta = 0$	*A*-1, *A*-2, *A*-3
B	1/20	35	Clamped; $\zeta = 0, V = 0$	Free; $Q = 0, M_\beta = 0$	*B*-1, *B*-2, *B*-3
C	1/10	35	Free; $Q = 0, M_\beta = 0$	Free; $Q = 0, M_\beta = 0$	*C*-1, *C*-2, *C*-3

The value of $\beta_1 = 35$ deg. was chosen to represent a reasonably sized hole. The ratio of 1/20 for $h' = h/r_1$ was selected so that direct comparison could be made with the results of the rotating cones (1) which were calculated for a comparable thickness. The value of $h' = 1/10$ for Case *C* was investigated as an indication of the effect of thickness. The

control plots, corresponding to Fig. 6, for each of the three cases are shown in Figs. 7, 8, 9. Solution points for each of a number of values of β_2 were obtained in the manner described with Fig. 6, and their locus gives a family of solutions each with the same boundary conditions.

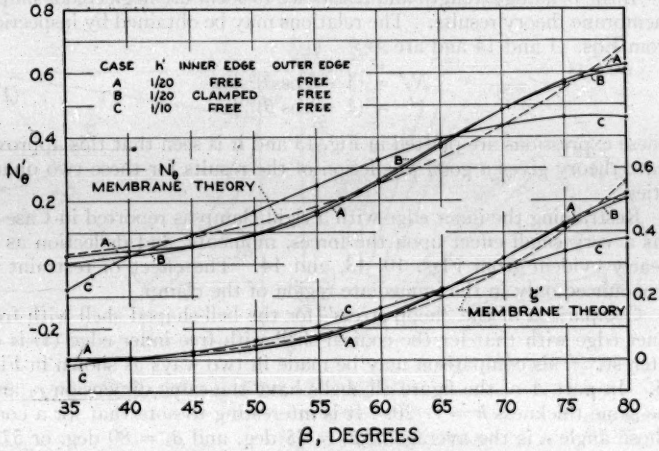


FIG. 13. Variation of "hoop stress" N_θ' and radial deflection ζ'.

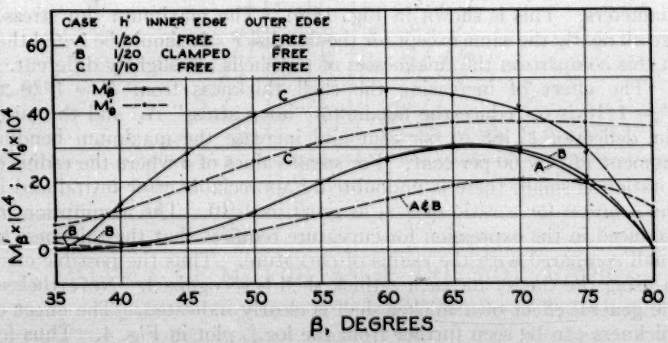


FIG. 14. Variation of moments M_β' and M_θ'.

Only the data for $\beta_2 = 80$ deg. were calculated. By linear interpolation and extrapolation from the three trial points in each case to the 80-deg. point the values of Q' and its derivatives were calculated. The values used in this calculation were read directly from the machine plotted curves for each of the trial solutions. The variations of M_β',

M_θ', N_θ', N_β', ζ' with β were next calculated from Eqs. 10 through 14. The resulting curves are shown in Figs. 10 through 14.

DISCUSSION

If the bending strength and resistance to shear are neglected a simple membrane theory results. The relations may be obtained by inspection from Eqs. 11 and 14 and are

$$\left.\begin{aligned} N_\theta' &= (1 - \cos\beta)^2 \\ \zeta' &= (1 - \cos\beta)^3. \end{aligned}\right\} \tag{18}$$

These expressions are plotted in Fig. 13 and it is seen that this approximate theory gives a good prediction of the results for these two quantities.

Restraining the inner edge with a rigid clamp as reported in Case *B* has a very small effect upon the forces, moments, and deflection as is clearly evident from Figs. 10, 13, and 14. The effect of restraint is pronounced only in the immediate region of the clamp.

Comparison of the "hoop stress" for the bell-shaped shell with free inner edge with that for the conical shell with free inner edge (1) is of interest. This comparison may be made in two ways as shown in Fig. 15. In part *A* of the figure all shells have the same dimension r_1 and the same thickness $h = r_1/20$. It is interesting to note that for a cone whose angle α is the average of $\beta_1 = 35$ deg. and $\beta_2 = 80$ deg. or 57.5 deg. the "hoop stress" at the outer rim is the same (5) as that for the bell. Comparison may also be made on the basis of equal outside diameters. This is shown in Fig. 15(*b*). The maximum rim stresses are all nearly the same except for the flat disk.[3] It should be noted that in this comparison the thicknesses of the shells are slightly different.

The effect of increasing the shell thickness from $h' = 1/20$ to $h' = 1/10$ is to reduce the maximum "hoop stress" N_θ' and the outer rim deflection ζ' by 26 per cent and increase the maximum bending moment M_β' by 60 per cent. For small values of β where the radius of rotation is small, there is undoubtedly appreciable error introduced in the solution for a ratio of h/r_1 as great as 1/10. The assumption introduced in the expression for curvature requires that the thickness be small compared with the radius of curvature. Thus the possible error in using the theory for such a thick shell is recognized. Nevertheless, the general effect of a thicker shell is clearly indicated. The effect of thickness can be seen further from the $\log_e j_0$ plot in Fig. 4. Thus for $\beta > 75$ deg. the differential equation does not depend on h'.

The j-coefficients in Eq. 9 approach infinite values very rapidly as β approaches 90 deg. This condition presented a difficulty in the

[3] It was shown in the discussion (3) of the conical shell investigation that the transition of stress values for cones to those for the flat rotating disk occurs abruptly as α changes from 83 to 90 deg.

analyzer operation for large values of β. If additional input equipment had been available it might have been possible to multiply each term in the equation by some function which would have eliminated or reduced the large values. Nevertheless, no trouble was experienced up to 80 deg. except for the computation of M_β' at this point where small

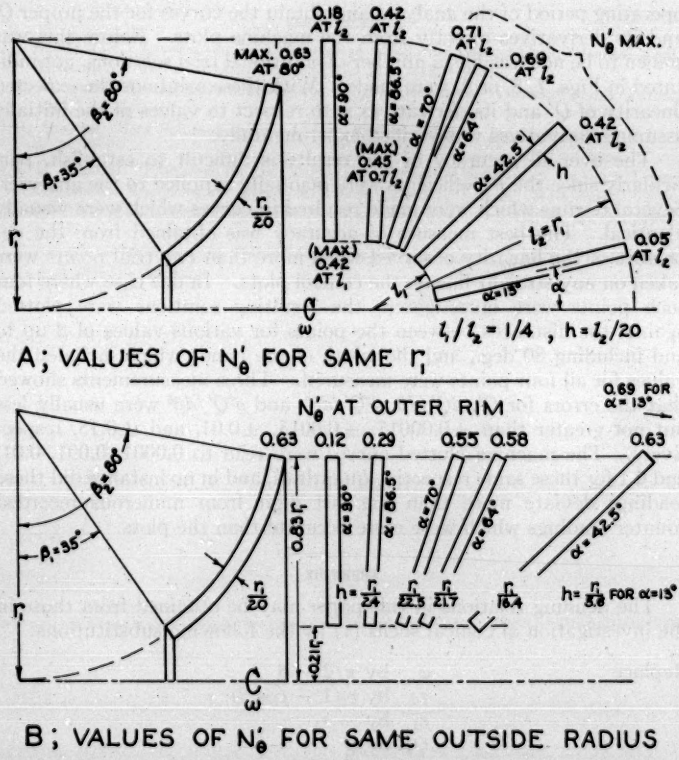


FIG. 15. Comparison of "hoop stresses" between bell-shaped shell and conical shells.

differences between large quantities began to assume importance. Uncertainty as to the location of the 80-deg. solution point was removed when additional solution points for lesser values of β_2 were constructed and the distances between these points measured along their locus were plotted against their values in degrees.

In Fig. 13 the slopes of the N_θ' curves are very small at $\beta_2 = 80$ deg.

and it may be inferred that for a shell of $\beta_2 = 90$ deg. there will be a negligibly small difference in maximum N_θ'. The portion of such a shell between $\beta_2 = 80$ deg. and $\beta_2 = 90$ deg. may be approximated by a flat disk or by a steep cone (1) in both of which cases the "hoop stress" N_θ' is decreasing with increased radial dimension.

The original intent was to determine the solution point during the operating period of the analyzer and obtain the curves for the proper Q' and its derivatives directly from the machine plots. Before this was shown to be not feasible, a number of additional trial solutions, not indicated in Figs. 7, 8, or 9, were made. With these solutions the expected linearity of Q' and its derivatives with respect to values of the initially assumed derivatives was verified experimentally.

The over-all accuracy of the results is difficult to establish, particularly since the j-coefficients were manually supplied to the analyzer. Several re-runs which were made resulted in curves which were visually identical. The best measure of accuracy was obtained from the departure of the linearity observed when more than two trial points were taken on any straight line on the control plots. In one case where four such points were investigated, the resulting solutions were plotted against the distances between the points for various values of β up to and including 80 deg., and the width of the bands which included the values for all four points were measured. These measurements showed that the errors for Q', $dQ'/d\beta$, $d^2Q'/d\beta^2$, and $d^3Q'/d\beta^3$ were usually less but not greater than ± 0.00015, ± 0.0015, ± 0.01, and ± 0.15, respectively. The machine plotted curves were read to 0.0001, 0.001, 0.01, and 0.1 for these same respective quantities, and in no instance did these readings deviate more than the last digit from numerous recorded counter readings which were more accurate than the plots.

APPENDIX

The defining relations in this paper may be obtained from those in the investigation of conical shells (1) by the following substitutions:

Replace		by
	φ	by $\pi/2 - \beta$
	r_0	by $r_1(1 - \cos\beta)$
	r_1	by $-r_1$
	Q_φ	by Q
	N_φ	by N_β
	M_φ	by M_β
	w	by $-w$
	e_φ	by e_β
	χ_φ	by χ_β

All other symbols remain the same.

Acknowledgment

The author wishes to acknowledge the helpful suggestions made by Professor P. L. Morton and the cooperation of the staff of the differential analyzer at the University of California at Los Angeles. The accurate checking and calculations of Mr. A. L. Hale are also acknowledged.

REFERENCES

(1) J. L. MERIAM, "Stresses and Displacements in a Rotating Conical Shell," *Journal Appl. Mechanics, Trans. A. S. M. E.*, Vol. 10, No. 2, p. A-53 (1943).

(2) H. WISSLER, "Festigkeitsberechnung von Ringflachenschalen," Doctoral Thesis, Zurich, 1917.

(3) K. STANGE, "Der Spannungszustand einer Kreisringschale," *Ingenieur Archiv*, Vol. 2, p. 47.

(4) J. L. MERIAM, "Procedure for the Machine or Numerical Solution of Ordinary Linear Differential Equations for Two-Point Linear Boundary Values," *Mathematical Tables and Other Aids to Computation*, The National Research Council, III, No. 28, Oct., 1949, p. 532.

(5) Discussion on Reference (1), *Journal Appl. Mechanics, Trans. A. S. M. E.*, Vol. 10, No. 4, p. A 233–234 (1943).

Anti-Lightning Seeding (*Weatherwise*, Vol. 3, No. 3).—At a recent scientific conference on thunderstorm electricity held at the University of Chicago, E. J. Workman, president of the New Mexico School of Mines, and S. E. Reynolds, project supervisor, told of experiments they had made which indicate that a seeding process, like that used to produce rain, may lessen the amount of electricity generated in clouds.

In thunderstorms moisture is carried into the upper level of the storm, where it freezes. And when water freezes, electricity is generated. In laboratory experiments chemicals were dropped into freezing water, and the New Mexico investigators were able to decrease the amount of electricity generated in the freezing process. Chemicals were also introduced into actual clouds during heavy rains on three occasions. In each instance the electrical generating powers of the thunderclouds were lessened.

Emergency Gas Supply from Stored Liquid (*Science News Letter*, Vol. 57 No. 23).—Emergency gas for heating and domestic fuel can be produced from liquids stored ready for instant use.

Designed to meet peak demands upon natural gas systems during the few days of severe winter weather, the emergency gas can be manufactured from air and some volatile petroleum hydrocarbon, as propane. A double overload can thus be met.

Dr. R. M. Deanesly of the Universal Oil Products Co., Chicago, reported to the American Chemical Society meeting in Houston that this autothermic process has been developed on a pilot plant scale and is ready for full scale application.

The air and propane are heated separately to over 1000° F., mixed, and allowed to react for a few hundredths of a second in a brick-lined stove. Some of the propane is burned and the rest is converted into smaller molecules. This gives gas that will mix with natural gas when fed into the system to tide over the emergency.

Electronic Umpire (*General Electric Review*, Vol. 53, No. 5).—An outstanding addition to one of baseball's training camps this year was a cross-eyed electronic umpire, which "calls 'em closer" than any normally endowed arbiter, and does so without fear of an argument. This fearless ruler of home plate was developed (for training use only) as an aid to improved pitching and batting, and operates even on an overcast day, but not at night.

In the thin metal ground box, upon which home plate is located, are lenses and mirrors which enable three electric eyes to "fence in" the strike zone by looking at the sky through three slots. When a ball passes though the strike zone, it casts its shadow on the electric eyes in a definite sequence, thereby creating electric impulses that light the strike-indicating lamp in a nearby cabinet.

An inside or outside ball does not cast its shadow on the eyes, because they "see" only the width of the plate. A high or low ball casts its shadow in an improper sequence and the lamp does not light, thus indicating a "ball." The equipment can be so adjusted that the strike zone outlined will fit any batter, regardless of his height.

Another attribute of this umpire is its ability to register the speed of a pitched ball.

RADIATION TRANSFER CONSIDERATIONS IN THE HEATING OF A CATHODE SLEEVE

BY

W. E. DANFORTH[1] and T. A. HADDAD[2]

ABSTRACT

The theory of radiation heat transfer is applied to the problem of heating a cathode sleeve to high temperatures. Experiments have been made in which metal and ceramic sleeves are heated by an internal cylindrical radiator. The results are discussed quantitatively in terms of the total emissivities of the several surfaces.

1. INTRODUCTION

The problem to be discussed here arose during experimentation with a type of electron tube cathode in which a ceramic sleeve is heated to the required emitting temperature by means of an internal heater. In the present stage of this development the ceramic sleeve is of thoria (with, perhaps, admixtures of other refractory substances) and the heater is a tungsten helix.

In order to obtain electron emission of many amperes per square centimeter from thoria, values of true surface temperature in the neighborhood of 1700° C. and higher must be attained.

At first sight, this seems to the designer an easy task inasmuch as the melting point of the tungsten heater is listed as 3400° C. In actuality, the temperature region 1700°–1800° C. appears to be a practical upper limit consistent with reasonable life of the tungsten heater.

If a cathode sleeve, in which a hole permits observations of the heater, is heated to 1800° C., one can see, making due allowances for uncertainties of the optical pyrometry, that the temperatures inside the sleeve can attain 2500° C. or more. Evaporation of tungsten at 2500° C. is such that short life would be expected.

The purpose of this investigation was to establish the cause of the large temperature difference between the heater and the outer surface of the sleeve and to formulate the physical principles involved.

2. GENERAL CONSIDERATIONS

Among those considerations which determine the sleeve temperature once the heater temperature is set, we may state the following: (*a*) Power is dissipated from the surface of the sleeve in an amount determined only by its outer temperature and the radiation constants.

[1] Assistant Director, Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

[2] Junior Physicist, Bartol Research Foundation, Swarthmore, Pa.

(*b*) Conditions must be such that the amount of power given by (*a*) is transferred from the heater to the inner surface of the sleeve, which transfer may involve conduction or radiation or both. In particular the temperature of the heater must exceed that of the sleeve by an amount necessary to effect this transfer. (*c*) There will be a temperature drop through the thickness of the sleeve in an amount consistent with its thermal conductivity.

When this investigation began, it was supposed that a large part of the temperature difference between heater and outer surface of the sleeve was due to low thermal conductivity of the latter. The experiments have shown, however, that the temperature drop in the sintered thoria sleeve is relatively small and that nearly the whole responsibility for the high heater temperature must be assigned to radiation-transfer considerations.

3. EXPERIMENTAL ARRANGEMENT

The primary experimental data on which this study is based were obtained with the arrangement shown in Fig. 1. A sintered thoria

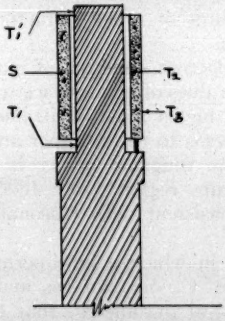


FIG. 1. Experimental arrangement. A ceramic sleeve *S* is heated by radiation from an internal molybdenum cylinder.

sleeve (*S*) was heated by radiation from an internal molybdenum cylinder which was maintained at high temperature by a furnace of the design shown in Fig. 2.

Observations of temperature of the outside of the sleeve (T_2) and of the top and bottom of the molybdenum rod (T_1 and T_1') were made with an optical pyrometer. Spectral emissivity values of 0.37 for molybdenum and 0.35 for thoria were used.

The thoria sleeves used all fitted loosely over the molybdenum rod, the average clearance being about 0.005 in. It was found to be relatively unimportant as regards heat transfer in the temperature region here concerned whether or not the sleeves are in contact at several

points. Therefore it was unnecessary to concern oneself with maintenance of symmetrical clearance between the rod and the sleeve.

The maximum temperature to which the projecting "heater" could be taken was about 1900° C. This required 75 amperes at 20 volts in the tungsten coil. Under these conditions, the lower section of the molybdenum heater (the part inside the tungsten coil of the furnace)

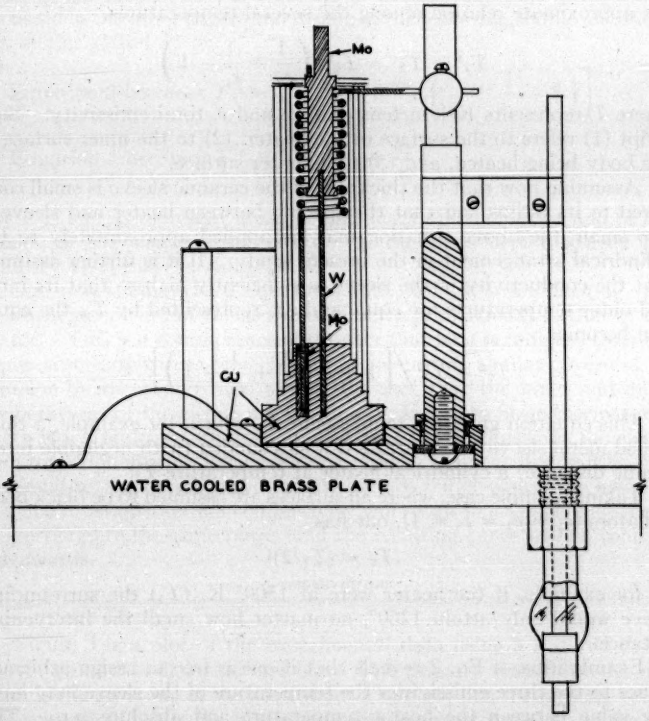


FIG. 2. Furnace arrangement, operated in a bell jar, for raising the molybdenum heater to the desired temperature.

was hot enough for recrystallization to occur, large grains being visible when the furnace was disassembled. Attempts to attain higher temperatures of the projecting molybdenum rod resulted in evaporation of molybdenum through the cracks in the furnace body.

The furnace was operated in a bell jar in which pressures in the region of 10^{-5} mm. Hg were maintained during the measurements.

A degree of vacuum as good as this was probably not required by the heat-transfer considerations. It was desirable however for long life and clean operation of the furnace unit.

4. MATHEMATICAL DISCUSSION

In another paper[3] it is shown that, in the case of the heating of a planar slab by a parallel planar heater, the following equation states the approximate relation among the several temperatures,

$$T_1^4 - T_2^4 = T_3^4 e_3 \left(\frac{1}{e_2} + \frac{1}{e_1} - 1 \right) \tag{1}$$

where T represents Kelvin temperature and e, total emissivity. Subscript (1) refers to the surface of the heater, (2) to the inner surface of the body being heated, and (3) to its outer surface.

Assuming now that the thickness of the ceramic sleeve is small compared to its radius and that the spacing between heater and sleeve is also small, the above equation may be applied approximately to the cylindrical arrangement in the present study. If it is further assumed that the conductivity of the sleeve is sufficiently high so that its inner and outer temperatures are equal and are represented by T_2, the equation becomes

$$T_2^4 = T_1^4 \Big/ \left[1 + e_3 \left(\frac{1}{e_2} + \frac{1}{e_1} - 1 \right) \right]. \tag{2}$$

This equation gives the temperature T_2 which, for example, a thin-walled metal sleeve would attain if it surrounded (with negligible intervening distance) a cylindrical heater at temperature T_1.

Taking a simple case, where all surfaces are assumed to be black body radiators ($e_1 = e_2 = e_3 = 1$), one has

$$T_2 = (T_1/2)^{\frac{1}{4}} \tag{3}$$

or, for example, if the heater were at 1500° K. (T_1) the surrounding sleeve would only attain 1260°, no matter how small the intervening distance.

Examination of Eq. 2 reveals that if one is free to assign arbitrary values to the three emissivities the temperature of the sleeve may have any value between the heater temperature and absolute zero. The values of the heater emissivity (e_1) and that of the inner surface of the sleeve (e_2) appear symmetrically in the expression and as either one approaches zero the sleeve temperature (T_2) approaches zero provided e_3 is finite. In the former case ($e_1 = 0$) no radiation is emitted by the heater; in the latter case ($e_2 = 0$) none is absorbed by the sleeve. As e_3 is permitted to become small the sleeve temperature T_2 increases since the rate of dissipation of energy by the sleeve is reduced. For the

[3] W. E. Danforth, JOURNAL FRANKLIN INSTITUTE, Vol. 250, No. 2, August, 1950, p. 146.

case of $e_3 = 0$, with e_1 and e_2 both finite, the sleeve has the same temperature as the heater.

In discussing the present data it has been found convenient to compare the observed sleeve temperatures with those to be expected for black body conditions ($e_1 = e_2 = e_3 = 1$). In making this calculation an approximate correction is made on account of the difference in radii of the heater and of the outer surface of the sleeve. This correction is discussed in the other paper[3] and results in the following simple modification of Eqs. 2 and 3:

$$\text{Equation 2 becomes } T_2^4 = T_1^4 \Big/ \left[1 + \frac{a_2}{a_1} e_3 \left(\frac{1}{e_2} + \frac{1}{e_1} - 1 \right) \right]$$
$$\text{Equation 3 becomes } T_2^4 = T_1^4 \Big/ \left[1 + \frac{a_2}{a_1} \right] \qquad (4)$$

where a_1 and a_2 represent area per unit length of the heater and of the outside of the sleeve, respectively. In the cases here discussed the quantity a_2/a_1 has the value 1.326.

One notes from these equations that, for radiation heating, the Kelvin temperature of the sleeve is directly proportional to that of the heater. This is a consequence of the fact that heat is radiated from the sleeve according to the *same power* of temperature as that governing its emission by the heater, and also of the fact that the inner and outer temperatures of the sleeve are the same. If, due to finite thermal conductivity, a temperature drop exists across the thickness of the sleeve the proportionality no longer obtains. Since the heat flux will be proportional to the fourth power of the surface temperature, the assumption of a constant thermal conductivity will result in a temperature drop proportional to the same power and the resulting curve will be concave downwards.

5. OBSERVATIONS

A. Uniform Sleeve

Figure 3 is a plot of the experimental data using a sintered thoria sleeve, ground both inside and out. The primary feature of the diagram is the grouping of the data about a straight line through the origin and somewhat below the black body line (referred to below as the BB line). This fact in itself may be taken as evidence that, in its main features, the loss of temperature between the heater and the outer surface of the sleeve is to be explained by radiation considerations rather than by poor thermal conductivity of the sleeve.

One notes that the first run, a series of ascending temperatures represented by the solid circles, is of different shape from the subsequent runs; its initial values lie above the BB line and the curve is concave upwards. These phenomena are believed to be the result of a slight

graying of the inner surface of the sleeve (initially pure white) which has the dual effect of (1) rendering the sleeve opaque to visible light[4] and (2) increasing, presumably, the total emissivity of the inner surface. By the end of the first run this matter seems to have reached equilibrium and subsequent runs follow the linear relationship within the experimental scatter. The observed slope of the line is 0.773, giving for example, a sleeve temperature of 1460° K. when the heater is

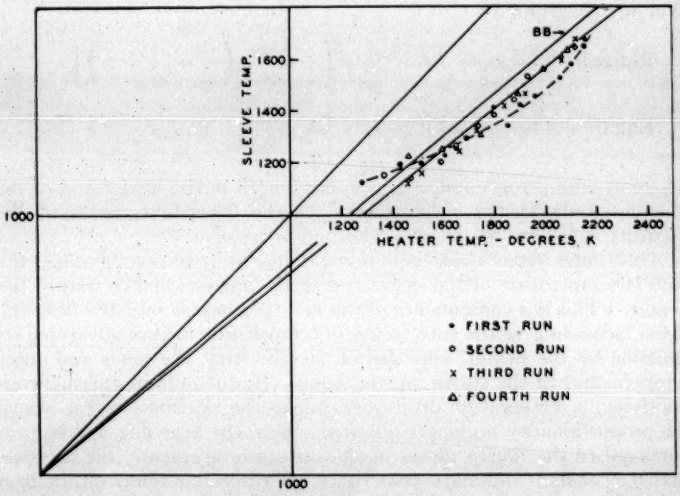


FIG. 3. Outer temperature of clean uniform thoria sleeve as a function of heater temperature.

1800° K. In accordance with the preceding section this requires that the several emissivities have values such that:

$$e_3\left(\frac{1}{e_2}+\frac{1}{e_1}-1\right)=1.374.$$

In the present case this is appropriately satisfied by using $e_1 = 0.22$ (in accordance with published values for molybdenum), $e_2 = 0.6$ for the darkened inner surface of the sleeve, and $e_3 = 0.26$ for its clean white outer surface.

[4] *Effect of transmitted light on pyrometry.* A clean thoria sleeve of 0.030 in. thickness, surrounding a heater whose temperature exceeds that of the sleeve by several hundred degrees, will transmit enough visible light to introduce large errors in the optical pyrometry. In the present work tests were run to determine the necessary corrections. Actually, it was found that the darkening of the inner surface, which always occurred in the first half hour of high temperature operation, was sufficient to make the correction unnecessary.

The latter value for the total emissivity of thoria is a very probable figure in the light of experience in this laboratory where values of this quantity between 0.26 and 0.36 were found.

The effect of increasing e_3 is shown by comparing Fig. 3 with Fig. 4 in which the same sleeve was given a blackened outer surface. (After obtaining the data of Fig. 4 the sleeve was removed from the apparatus and given a thin coating of grease which was subsequently carbonized by heating in vacuum.) It is seen that the outer temperatures are considerably reduced. Assuming that e_1 and e_2 are unchanged from the

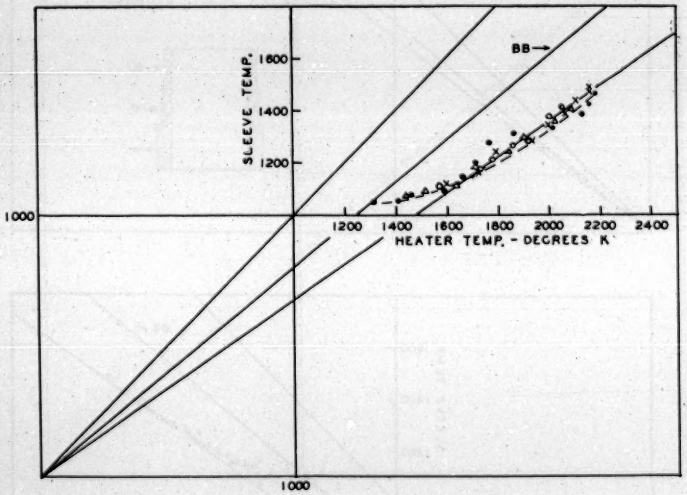
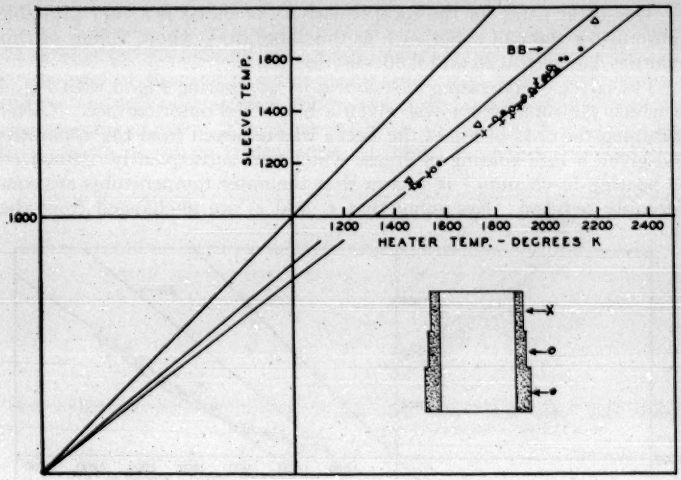


FIG. 4. Outer temperature of darkened thoria sleeve as a function of heater temperature.

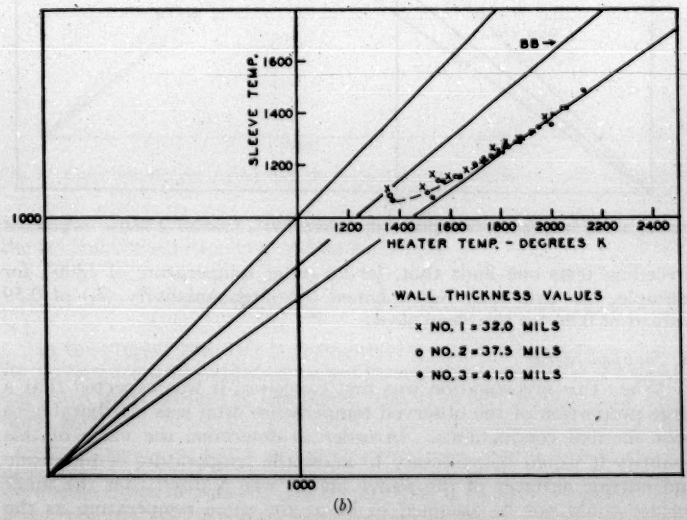
preceding tests one finds that, for an outer temperature of 1300°, for example, the data may be explained by outer emissivity (e_3) of 0.59 instead of 0.26 for the clean sleeve.

B. Stepped Sleeve

When this investigation was first conceived it was expected that a large proportion of the observed temperature drop was attributable to poor thermal conductance. In order to determine the value of this quantity it would be necessary to know the temperature of the inside and outside surfaces of the sleeve and it was realized that the inner surface could not be assumed to be at the same temperature as the heater. In view of this difficulty sleeves were made as shown in the insert on Fig. 5(*a*) with three longitudinal sections having the same



(*a*)



(*b*)

FIG. 5. Outer temperature *versus* heater temperature for sleeves having three different wall thicknesses. Note lack of effect of wall thickness.

inside diameters but different wall thicknesses. The value of thermal conductivity was to be deduced from observations of the different outside temperatures of the three sections.

Figure 5(*a*) shows data taken with the stepped sleeve. It would be expected, on the basis of thermal conductivity, that the thinnest section shown by the crosses would show considerably higher temperatures than the thickest section shown by the solid circles. Inspection of the diagram shows that no correlation between temperature and sleeve thickness is observed. If one assumes that conductivity is the

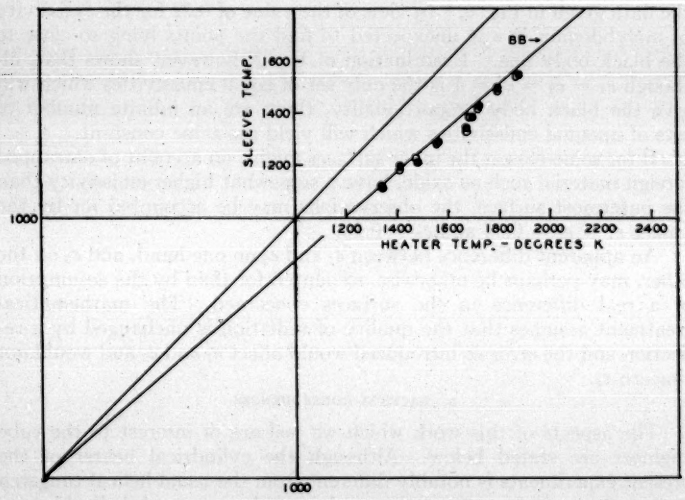


FIG. 6. Temperature of metal sleeve as a function of heater temperature. The specific radiation transfer conditions have resulted in agreement with black body predictions even though all emissivities are far below unity.

determining circumstance, and that the inner surface of the sleeve is at the temperature of the heater, it is possible, by a graphical solution of a fourth power equation[5], to compute the difference to be expected between the thinnest and the thickest sections. To do this it is necessary to assume that the thermal conductivity increases with temperature in such a manner as to give the observed proportionality. The results of such a calculation are shown on Fig. 5(*a*) in which, assuming that the line through the points gives the temperature of the thickest section,

[5] The calculation must recognize the fact that, since a thinner section is at a higher temperature, the flux of radiant heat through it will be greater. This destroys the linear relationship between temperature and thickness which, at first glance, might be expected to exist.

the triangles give the calculated temperature of the thinnest section. It is evident that the thermal conductance assumption is inconsistent with the observations. Values of thermal conductivity which had to be assumed in this calculation lie between 1.7×10^{-4} and 7.2×10^{-4}. The three wall thicknesses are 0.032, 0.037 and 0.041 in. Results of another test with a stepped sleeve are plotted in Fig. 5(*b*) and show essentially the same pattern.

C. Metal Sleeve

Replacement of the ceramic sleeve by a molybdenum sleeve yields the data given in Fig. 6. In view of the value of 0.22 for the emissivity of molybdenum it was unexpected to find the points lying so close to the black body line. Examination of Eq. 2, however, shows that, although $e_1 = e_2 = e_3 = 1$ is the only set of equal emissivities which will give the black body proportionality, there are an infinite number of sets of unequal emissivities which will yield the same constant.

If for some reason the inner surfaces might, on account of entrapped foreign material such as oxide, have a somewhat higher emissivity than the outermost surface, the observations may be accounted for by the values $e_1 = e_2 = 0.33$ and $e_3 = 0.20$.

An apparent difference between e_1 and e_2 on one hand, and e_3 on the other, may perhaps be otherwise accounted for than by the assumption of a real difference in the surfaces concerned. The mathematical treatment assumes that the quality of radiation is unchanged by a reflection and the error so introduced would affect e_1 and e_2 and would not concern e_3.

6. PRACTICAL CONSEQUENCES

The aspects of this work which we feel are of interest to the tube engineer are stated below. Although the cylindrical heater of the present experiments is notably different from the usual helical tungsten heater, the main features of the results are the same. A helical heater may be considered as a cylindrical heater with a fictitious value of emissivity. Whether its effective emissivity will be greater or less than the intrinsic emissivity of the tungsten will depend on size of wire, pitch, etc. Obviously the heater can never be more effective than a black body, so $e_1 \leqq 1$.

In any example of heating a cathode sleeve it is likely that the line of T_2 *vs.* T_1 will lie between the black body line and the especially unfavorable situation shown in Fig. 4. In other words, we find by extrapolation of the figure that, if a sleeve is to be heated to 1800° C., the heater temperature must be at least 2300° C. and may even have to be as high as 2800° C. if unfavorable but not impossible emissivities are encountered.

Thus 1800° C. appears on rather basic grounds to be an upper limit to which a sleeve may be heated by a tungsten coil, unless, by some

means not now apparent, the emissivity of the outer surface of the sleeve can be reduced to such a low value that the temperature data lie above the black body line. Values of total emissivity as low as 0.1 will be necessary to give substantial improvement.

These remarks apply either to ceramic or metal sleeves. Although it might be thought that the lower thermal conductivity of ceramic would result in a large temperature drop this was found not to be true of the sintered thoria sleeves.[6]

[6] Unpressed pure thoria ceramic of density between 5 and 7 g./cm.^{3}.

RADIATION HEATING OF A PLANE PARALLEL SLAB BY A PARALLEL SEMI-INFINITE HEATER

BY

W. E. DANFORTH[1]

ABSTRACT

The problem is considered where a heater of emissivity e_1, is maintained at a temperature T_1. The slab being heated has emissivity and equilibrium temperature e_2 and T_2 on its inner surface and e_3 and T_3 on its outer surface. The equation is developed which relates the three temperatures in terms of the three emissivities.

1. INTRODUCTION

In another paper (1)[2] experiments are described concerning the heating of a cylindrical sleeve by radiation from an internal cylinder. In the present paper the mathematical basis is developed for the interpretation of these experiments. The previous writings on this subject, which have come to the author's attention, are those of Meissner and Voigt (2) who give a basic theoretical and experimental treatment of heat-transfer considerations in a dewar flask and of Heinze and Wagener (3) who present a general theory of radiation transfer using formulation principles which are equivalent to those employed herein but which, according to the authors, permit more facile solutions of complex problems. The present paper treats only that problem of the simplest possible geometry and attempts to present, in a manner which will be clear and compelling to those previously unacquainted with this subject, the basic physical picture of heat transfer in vacuum.

2. PRELIMINARY REMARKS

This problem is considered in its equilibrium case. Considering Fig. 1, the body II is being heated by radiation from body I. Temperatures of the three surfaces concerned are represented by T_1, T_2 and T_3; total emissivities by e_1, e_2, and e_3. Energy is radiated from surface 3 into empty space in the amount $\sigma e_3 T_3^4$ where σ is the Stefan-Boltzman constant. Both bodies are assumed opaque to all frequencies of the radiation concerned. Any difference between T_3 and T_2 is to be ascribed to finite thermal conductivity of body II. It is assumed that the spectral distribution of the radiation is unchanged upon reflection from any surface.

Energy flux leaving surface 3 of body II is simply described as

[1] Assistant Director, Bartol Research Foundation of The Franklin Institute, Swarthmore, Pa.

[2] The boldface numbers in parentheses refer to the references appended to this paper.

primary thermal radiation originating in body II and flowing into empty space. No energy crosses 3 from the space. At surfaces 1 and 2, however, the situation is more complex. In addition to primary radiation from the body concerned, there exists additional energy flux resulting from multiple reflections of radiation having its origin in both bodies.

3. METHOD OF CALCULATION

The problem is explicitly stated as follows. Given the values of emissivity and given T_3, the temperature of the outermost surface, what must be the values of T_1 and T_2 in order that a net flux $E_0 = \sigma e_3 T_3^4$ exist across any plane to the right of body I?

Since body II is opaque to radiation, T_2 exceeds T_3 by an amount required to cause the flux E_0 to be transported by simple conduction. In other words:

$$T_3 - T_2 = \frac{t}{k}\sigma e_3 T_3^4 \tag{1}$$

where k is the thermal conductivity, and t is the thickness of the slab.

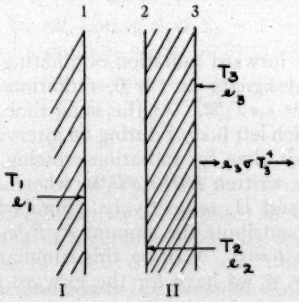


FIG. 1.

What, now, must be the value of T_1 in order that the flux across a plane in the space between the bodies be also equal to E_0?

One sees immediately that the flux from left to right in the interspace, involving as it does reflected radiation, some of which originated in body II, must involve temperatures of both bodies, *viz.*, T_1 and T_2. The method to be employed here will be as follows:

1. The total forward radiation in the interspace (energy flux from left to right) will be computed by considering the primary radiation from bodies I and II and the summative effect of all reflections.
2. The backward radiation will be computed in like manner.

3. The net forward radiation is then equated to E_0 resulting in a functional relation among T_1, T_2, and T_3, involving the corresponding emissivities.

Since we are concerned with the equilibrium case, time will not appear in the result. As a logical scaffolding, however, it has been found convenient to speak in terms of time in order to attain clarity in formulating the summations.

Proceeding with (1) we recognize that the forward or the backward flux at any given time is the resultant of contributions originating at different times in the past, and existing at the given time as a result of multiple reflections. Therefore, to compute the radiation at a given time it is necessary to carry out a summation of contributions which originate at times extending to the infinite past.

We proceed to compute the energy flux across a plane in the interspace. By symmetry the position of this plane is immaterial so for convenience we place it at zero distance from body I. Considering a time element δt, the energy which flows across the plane during this interval may be divided into two categories: (*a*) radiation originating at body I, and (*b*) radiation originating at body II.

Forward Radiation

We first consider forward radiation originating at I. At any given instant, which we designate as $t = 0$, radiation from I contributed energy in the amount $e_1\sigma T_1^4\delta t$. At the same time, however, additional energy is present which left body I during an interval δt at time $t = -2\tau$ where τ is the transit time for radiation crossing the interface. This contribution may be written $R_1R_2e_1\sigma T_1^4\delta t$, where R_1 and R_2 are reflectivities of bodies I and II, respectively. Similarly, the interval δt at time $t = -4\tau$ will contribute the amount $R_1^2R_2^2e_1\sigma T_1^4$ to the radiation emitted during δt at $t = 0$. Writing this summation for all contributions back to $t = -\infty$ we have for the forward radiation originating at I:

$$e_1\sigma T_1^4\{1 + R_1R_2 + R_1^2R_2^2 + \cdots\}\delta t = \frac{e_1\sigma T_1^4}{1 - R_1R_2}\delta t. \qquad (2)$$

We now consider the forward radiation originating at II. It is evident that no primary radiation having its origin at $t = 0$ contributes to δt at $t = 0$. The contribution of most recent origin left body II at time $t = -\tau$ and crosses our plane in the forward direction at $t = 0$ with intensity $R_1e_2\sigma T_2^4\delta t$. At time $= -3\tau$ another contribution originated and arrives with intensity $R_1^2R_2e_2\sigma T_2^4\delta t$. Proceeding as above we have:

$$e_2\sigma T_2^4R_1\{1 + R_1R_2 + R_1^2R_2^2 + \cdots\}\delta t = \frac{e_2\sigma T_2R_1}{1 - R_1R_2}\delta t. \qquad (3)$$

Backward Radiation

Now computing (2), the backward radiation, we consider first the contributions originating at body I. It will be seen that the contributions during δt at $t = 0$ have resulted from primary radiation emitted at times $t = -\tau$, $t = -3\tau$, $t = -5\tau$, etc. Carrying out the sum we write for this case

$$e_1\sigma T_1^4 R_2\{1 + R_1R_2 + R_1^2R_2^2 + \cdots\}\delta t = \frac{e_1\sigma T_1^4 R_2}{1 - R_1R_2}. \tag{4}$$

Finally, for the backward radiation originating at II we proceed similarly and obtain the sum

$$e_2\sigma T_2^4\{1 + R_1R_2 + R_1^2R_2^2 + \cdots\}\delta t = \frac{e_2\sigma T_2^4}{1 - R_1R_2}\delta t. \tag{5}$$

Now proceeding with step (3) we equate the difference between the forward radiation (2) + (3) and the backward radiation (4) + (5) to the equilibrium flux $e_3\sigma T_3^4\delta t$ and obtain:

$$\frac{\sigma\delta t}{1 - R_1R_2}\{e_1T_1^4 + R_1e_2T_2^4 - R_2e_1T_1^4 - e_2T_2^4\} = e_3\sigma T_3^4\delta t. \tag{6}$$

Dividing through by $\sigma\delta t$, noting that $R_2 = 1 - e_2$, and reducing the resulting equation to its simplest algebraic form, one has

$$\frac{T_1^4 - T_2^4}{\frac{1}{e_2} + \frac{1}{e_1} - 1} = e_3T_3^4. \tag{7}$$

4. APPLICATION

The results of this calculation are applied, in another paper (1), to the case of the heating of a ceramic sleeve by an internal cylindrical radiator. In the case of a sleeve whose wall thickness is small compared to the diameter and where the diameter of the heater is approximately equal to the inside diameter of the sleeve, the above calculation for the parallel plane case may be applied. If one further assumes that the sleeve is a sufficiently good conductor so that the temperature drop over its wall thickness may be neglected, one writes $T_3 = T_2$, and the relation between sleeve temperature and heater temperature may be written

$$T_2^4 = T_1^4 \Big/ \left[1 + e_3\left(\frac{1}{e_2} + \frac{1}{e_1} - 1\right)\right]. \tag{8}$$

In the case where a sleeve fits a cylindrical heater closely as far as the inside diameter is concerned, but where the wall thickness of the sleeve is not negligible, a simple modification of the above equations is to be made. Again it is assumed that the temperature drop in the sleeve may be neglected.

Let a_1 = surface area of body I, and area of the inside surface of body II, per unit length.

a_2 = area of outside surface of body II, per unit length.

Examination of the foregoing derivation reveals that Eqs. 1–5 are unchanged, requiring only multiplication by a_1 to apply to the present case. In writing Eq. 6, however, it will be recognized that, although the left hand side is to be multiplied by a_1, the right hand side (representing heat radiated from the outermost surface) should be multiplied by a_2. Doing this, one sees at once that the result is simply to change the emissivity e_3 by the factor $\frac{a_2}{a_1}$. One therefore has, in place of (8), the result:

$$T_2^4 = T_1^4 \Big/ \left[1 + \frac{a_2}{a_1} e_3 \left(\frac{1}{e_2} + \frac{1}{e_1} - 1 \right) \right]. \tag{9}$$

In the case where all surfaces are black bodies, this reduces to

$$T_2^4 = T_1^4 \Big/ \left(1 + \frac{a_2}{a_1} \right).$$

5. CRITICISM OF METHOD

In the above development the radiation is treated as a one-dimensional phenomenon such as a plane wave originating at the surface of body I. Actually it is realized that the emission from any element of surface of body I occurs in all directions of a hemisphere and hence could logically receive three dimensional treatment as is done by Meissner & Voigt (2). Indeed, such treatment is essential in a theoretical discussion where the variation of reflectivity with angle of incidence is explicitly recognized. The present treatment, however, was carried out for the purpose of interpreting certain experimental results (1) in which reflectivity figures are derived from empirical hemispherical emissivity data. We assume, in effect, that this empirical reflectivity pertains to all angles of reflection and this necessarily introduces an element of approximation.

The other major element of approximation is the assumption that the same effective value of reflectivity applies to all successive reflections of a given element of radiated energy. In other words, it is assumed that the reflectivity is independent of wave-length. The nearer e_1 and e_2 approach unity, the smaller will be the error introduced by this assumption.

REFERENCES

(1) W. E. DANFORTH AND T. A. HADDAD, JOUR. FRANKLIN INSTITUTE, Vol. 250, No. 2, August, 1950, p. 135.

(2) W. MEISSNER AND B. VOIGT, *Zeits. f. Instrumentkunde*, Vol. 50, p. 121 (1930).

(3) W. HEINZE AND S. WAGENER, *Zeits. f. tech. phys.*, Vol. 18, p. 75 (1937).

INTERFLECTIONS IN COUPLED ENCLOSURES

BY

PARRY MOON[1] AND DOMINA EBERLE SPENCER[2]

The classical treatment of photometrics is entirely inadequate to meet modern needs because it neglects interflections and considers only direct light from the sources. H. Buckley made a great advance in 1927, when he showed that the interflections of light within an enclosure can be formulated by means of an integral equation (1).[3] The integral equations for a number of cases have been solved (2) and the results have been tabulated (3).

All previous work, however, has been limited to a single enclosure. We now extend the theory to two enclosures that are coupled optically by a flat plate of translucent material.

The problem arose in the design of modern room lighting with luminous ceilings made of diffusing material and lighted from above by fluorescent lamps. In the older methods of lighting, there is the problem of interflections within the room and there is the problem of interflections within the luminaire. But, because of the great disparity between room area and luminaire area, the two problems can be considered separately. With a luminous ceiling, however, the area of the luminaire is the same as the floor area; and any change in the room reflectances will affect the amount of light from the luminaire. The two interflection problems merge into one.

This practical problem gave the impetus to the development of interflection theory for coupled systems, but the new theory will probably have other applications. The following sections treat of the general question of interflections in two coupled enclosures. Several special cases are then considered, and equations and graphs are given for room lighting.

1. THE GENERAL PROBLEM

Two enclosures, S_1 and S_2 of Fig. 1, are coupled by a flat, transmitting plate S. This plate is of diffusing material and has a uniform reflectance ρ and transmittance τ, when measured apart from the enclosures and with a perfectly absorbing backing.

Coordinate systems are associated with the surfaces so that a point on S is specified by (s, t), a point on S_1 is designated by (s_1, t_1), and a point on S_2 is given by (s_2, t_2). The surfaces S_1 and S_2 reflect light; and their reflectances are, in general, functions of position: $\rho_1(s_1, t_1)$, $\rho_2(s_2, t_2)$.

[1] Massachusetts Institute of Technology, Cambridge, Mass.

[2] Brown University, Providence, R. I.

[3] The boldface numbers in parentheses refer to the references appended to this paper.

In the general case, the helios[4] also varies with position on the surfaces. The values of helios for S_1 and S_2, including all interflections, are designated as $H_1(s_1, t_1)$ and $H_2(s_2, t_2)$. The helios of the transmitting plate is $H_3(s, t)$ as viewed from S_1, and $H_4(s, t)$ as viewed from S_2. All surfaces are assumed to be perfectly diffusing.

The helios $H_3(s, t)$ is produced by both incident and transmitted light, so

$$\left.\begin{aligned} H_3(s, t) &= \rho D_3(s, t) + \tau D_4(s, t), \\ H_4(s, t) &= \rho D_4(s, t) + \tau D_3(s, t), \end{aligned}\right\} \tag{1}$$

where D_3 and D_4 are the pharosage (lumen m^{-2}) incident on the two sides of the transmitting plate.

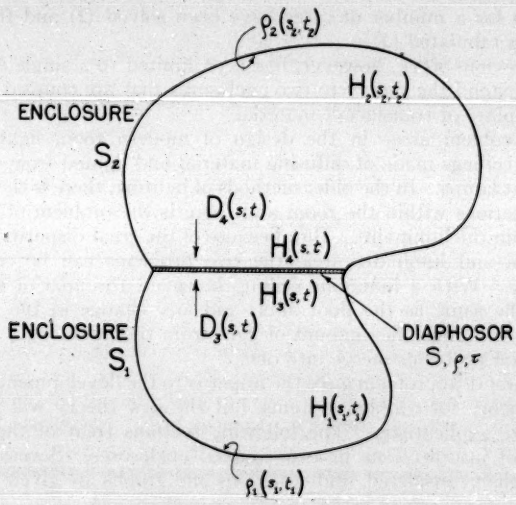


FIG. 1. General case of two coupled enclosures.

As in the previous work (3), the pharosage can be expressed in terms of integral equations of the Fredholm type:

$$\left.\begin{aligned} D_3(s, t) &= \frac{1}{\rho} H_{03}(s, t) + \int_{S_1} K_{13}(s, t; u, v)\, H_1(u, v)\, du\, dv, \\ D_4(s, t) &= \frac{1}{\rho} H_{04}(s, t) + \int_{S_2} K_{24}(s, t; u, v)\, H_2(u, v)\, du\, dv. \end{aligned}\right\} \tag{2}$$

Here H_{03} is the helios without interflections, produced by direct light from lamps enclosed in S_1, while H_{04} is the helios caused by direct light

[4] Or generalized brightness. The nomenclature of this paper is the same used in Reference 3.

from lamps in S_2. The kernel K_{ij} is equal to the incident pharosage produced on surface j by light from an element of surface i, per unit helios, per unit $(du\ dv)$ at i.

Substitution of Eq. 2 into 1 gives

$$H_3(s,t) = H_{03}(s,t) + \rho \int_{S_1} K_{13}(s,t;u,v)\, H_1(u,v)\, du\, dv + \frac{\tau}{\rho} H_{04}(s,t) + \tau \int_{S_2} K_{24}(s,t;u,v)\, H_2(u,v)\, du\, dv,$$

$$H_4(s,t) = H_{04}(s,t) + \rho \int_{S_2} K_{24}(s,t;u,v)\, H_2(u,v)\, du\, dv + \frac{\tau}{\rho} H_{03}(s,t) + \tau \int_{S_1} K_{13}(s,t;u,v)\, H_1(u,v)\, du\, dv.$$

Similar integral equations express the helios H_1 and H_2. Thus the general problem of interflections in two coupled enclosures is characterized by the four integral equations:

$$H_1(s_1,t_1) = H_{01}(s_1,t_1) + \rho_1(s_1,t_1) \left\{ \int_{S_1} K_{11}(s_1,t_1;u,v)\, H_1(u,v)\, du\, dv + \int_{S} K_{31}(s_1,t_1;u,v)\, H_3(u,v)\, du\, dv \right\},$$

$$H_2(s_2,t_2) = H_{02}(s_2,t_2) + \rho_2(s_2,t_2) \left\{ \int_{S_2} K_{22}(s_2,t_2;u,v)\, H_2(u,v)\, du\, dv + \int_{S} K_{42}(s_2,t_2;u,v)\, H_4(u,v)\, du\, dv \right\}, \tag{3}$$

$$H_3(s,t) = H_{03}(s,t) + \frac{\tau}{\rho} H_{04}(s,t) + \rho \int_{S_1} K_{13}(s,t;u,v)\, H_1(u,v)\, du\, dv + \tau \int_{S_2} K_{24}(s,t;u,v)\, H_2(u,v)\, du\, dv,$$

$$H_4(s,t) = H_{04}(s,t) + \frac{\tau}{\rho} H_{03}(s,t) + \rho \int_{S_2} K_{24}(s,t;u,v)\, H_2(u,v)\, du\, dv + \tau \int_{S_1} K_{13}(s,t;u,v)\, H_1(u,v)\, du\, dv.$$

The kernels depend only on the geometry of the surfaces and can be found by elementary methods (2). The values of reflectance and transmittance are evaluated experimentally for the particular materials used in the enclosures, while H_{oi} are calculated from the known characteristics of the light sources. There remain only the four unknowns H_1, H_2, H_3, and H_4, which can be found by simultaneous solution (5) of the four equations 3.

2. SIMPLIFIED CASE FOR INFINITE CYLINDERS

The solution of Eq. 3 is difficult, largely because helios is a function of two independent variables (4). A great simplification is effected if helios is expressible as a function of a single variable. For example, suppose that S_1 and S_2 of Fig. 1 are cylinders of infinite length in the direction perpendicular to the page, and that lighting is uniform in this direction. For simplicity, we take the usual case where $\rho_1 = \text{const}$ and $\rho_2 = \text{const}$. Then Eq. 3 becomes

$$H_1(s_1) = H_{01}(s_1) + \rho_1 \left\{ \int_{S_1} K_{11}(s_1, t)\, H_1(t)\, dt + \int_S K_{31}(s_1, t)\, H_3(t)\, dt \right\},$$

$$H_2(s_2) = H_{02}(s_2) + \rho_2 \left\{ \int_{S_2} K_{22}(s_2, t)\, H_2(t)\, dt + \int_S K_{42}(s_2, t)\, H_4(t)\, dt \right\}, \tag{4}$$

$$H_3(s) = H_{03}(s) + \frac{\tau}{\rho} H_{04}(s) + \rho \int_{S_1} K_{13}(s, t)\, H_1(t)\, dt + \tau \int_{S_2} K_{24}(s, t)\, H_2(t)\, dt,$$

$$H_4(s) = H_{04}(s) + \frac{\tau}{\rho} H_{03}(s) + \rho \int_{S_2} K_{24}(s, t)\, H_2(t)\, dt + \tau \int_{S_1} K_{13}(s, t)\, H_1(t)\, dt.$$

3. COUPLED CYLINDERS OF FINITE LENGTH

A case of particular importance occurs with a cylinder of finite length (Fig. 2), divided into two parts by a diaphosor or diffusing plate, and having plane parallel ends. With a circular cylinder, because of symmetry, the helios is uniform at any given depth in the cylinder. With other cross-sections, one may work with *average* helios at a given depth and may assume that the helios is uniform on the top and bottom. In this way, an approximate solution, close enough for practical purposes (3), is obtained without resorting to the two-dimensional helios variation of Eq. 3.

In Fig. 2, the reflectance is uniform over each surface. The diaphosor has reflectance ρ and transmittance τ, as in Section 1. Average helios of the ends are H_3 and H_5, while the average helios of the diaphosor is H_2 as seen from below and H_6 as seen from above. The average helios of the cylindrical walls at relative depth s or s' is called $H_1(s)$ or $H_4(s')$, where

$$s = u/w, \qquad t = v/w,$$
$$s' = u'/w, \qquad t' = v'/w,$$

and w is a standard width which is arbitrarily taken as norm (3).

Corresponding to Eq. 1, we now have

$$\left.\begin{aligned} H_2 &= \rho D_2 + \tau D_6, \\ H_6 &= \rho D_6 + \tau D_2. \end{aligned}\right\} \tag{5}$$

But

$$D_2 = \frac{H_{02}}{\rho} + \int_0^{h/w} K_{12}(t)\, H_1(t)\, dt + K_{32} H_3,$$

$$D_6 = \frac{H_{06}}{\rho} + \int_0^{h'/w} K_{46}(t')\, H_4(t')\, dt' + K_{56} H_5.$$

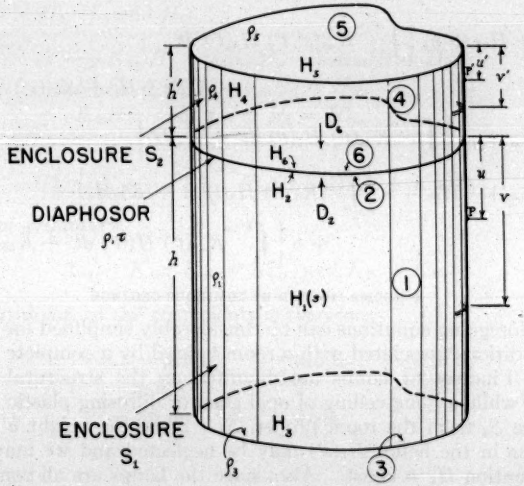


FIG. 2. Coupled cylinders. The cross-section may have any shape, including rectangular.

The kernels K_{ij} are evaluated in a previous paper (5). The helios H_{oi} is that obtained directly from the lamps, without interflections and without transmission through the diaphosor. Then Eq. 5 becomes

$$H_2 = H_{02} + \frac{\tau}{\rho} H_{06} + \rho \left\{ \int_0^{h/w} K_{12}(t)\, H_1(t)\, dt + K_{32} H_3 \right\} + \tau \left\{ \int_0^{h'/w} K_{46}(t')\, H_4(t')\, dt' + K_{56} H_5 \right\},$$

$$H_6 = H_{06} + \frac{\tau}{\rho} H_{02} + \tau \left\{ \int_0^{h/w} K_{12}(t)\, H_1(t)\, dt + K_{32} H_3 \right\} + \rho \left\{ \int_0^{h'/w} K_{46}(t')\, H_4(t')\, dt' + K_{56} H_5 \right\}.$$

The complete set of equations for the coupled cylinders is

$$
\begin{aligned}
H_1(s) &= H_{01} + \rho_1 \left\{ \int_0^{h/w} K_{11}(s,t)\, H_1(t)\, dt + K_{21}(s) \cdot H_2 + K_{31}(s) \cdot H_3 \right\}, \\
H_2 &= H_{02} + \frac{\tau}{\rho} H_{06} + \rho \left\{ \int_0^{h/w} K_{12}(t)\, H_1(t)\, dt + K_{32} H_3 \right\} \\
&\qquad + \tau \left\{ \int_0^{h'/w} K_{46}(t')\, H_4(t')\, dt' + K_{56} H_5 \right\}, \\
H_3 &= H_{03} + \rho_3 \left\{ \int_0^{h/w} K_{13}(t)\, H_1(t)\, dt + K_{23} H_2 \right\}, \\
H_4(s') &= H_{04} + \rho_4 \left\{ \int_0^{h'/w} K_{44}(s',t')\, H_4(t')\, dt' \right. \\
&\qquad \left. + K_{54}(s') \cdot H_5 + K_{64}(s') \cdot H_6 \right\}, \\
H_5 &= H_{05} + \rho_5 \left\{ \int_0^{h'/w} K_{45}(t')\, H_4(t')\, dt' + K_{65} H_6 \right\}, \\
H_6 &= H_{06} + \frac{\tau}{\rho} H_{02} + \tau \left\{ \int_0^{h/w} K_{12}(t)\, H_1(t)\, dt + K_{32} H_3 \right\} \\
&\qquad + \rho \left\{ \int_0^{h'/w} K_{46}(t')\, H_4(t')\, dt' + K_{56} H_5 \right\}.
\end{aligned}
\tag{6}
$$

4. ROOMS LIGHTED BY LUMINOUS CEILINGS

The foregoing equations can be considerably simplified for the practical conditions associated with a room lighted by a complete luminous ceiling. Fluorescent lamps are mounted on the structural ceiling *5* (Fig. 2), while a hung ceiling of opal glass or diffusing plastic separates the space S_2 from the room proper, S_1. Since the height h' is small, variations in the helios $H_4(s')$ may be neglected and we may use the approximation $H_4 = \text{const.}$ Also, since the lamps are all contained in S_2, the helios H_{01}, H_{02}, and H_{03} are zero.

With these simplifications, Eq. 6 becomes

$$
\begin{aligned}
H_1(s) &= \rho_1 \left\{ \int_0^{h/w} K_{11}(s,t)\, H_1(t)\, dt + K_{21}(s) \cdot H_2 + K_{31}(s) \cdot H_3 \right\}, \\
H_2 &= \frac{\tau}{\rho} H_{06} + \rho \left\{ \int_0^{h/w} K_{12}(t)\, H_1(t)\, dt + K_{32} H_3 \right\} \\
&\qquad + \tau \{ K_{46} H_4 + K_{56} H_5 \}, \\
H_3 &= \rho_3 \left\{ \int_0^{h/w} K_{13}(t)\, H_1(t)\, dt + K_{23} H_2 \right\}, \\
H_4 &= H_{04} + \rho_4 \{ K_{44} H_4 + K_{54} H_5 + K_{64} H_6 \}, \\
H_5 &= H_{05} + \rho_5 \{ K_{45} H_4 + K_{65} H_6 \}, \\
H_6 &= H_{06} + \tau \left\{ \int_0^{h/w} K_{12}(t)\, H_1(t)\, dt + K_{32} H_3 \right\} + \rho \{ K_{46} H_4 + K_{56} H_5 \}.
\end{aligned}
\tag{7}
$$

Here, since $H_4 = \text{const}$, H_4 is taken outside the integrals of Eq. 6 and

$$K_{45} = \int_0^{h'/w} K_{45}(t')\, dt',$$

$$K_{46} = \int_0^{h'/w} K_{46}(t')\, dt',$$

$$K_{44} = K_{44}(h'/2w) = \int_0^{h'/w} K_{44}(h'/2w, t')\, dt'.$$

The last three equations of (7) may be considered as a set of linear equations in H_4, H_5, and H_6:

$$\left.\begin{aligned} &(1 - \rho_4 K_{44})H_4 - \rho_4 K_{54} H_5 - \rho_4 K_{64} H_6 = H_{04}, \\ &-\rho_5 K_{45} H_4 + H_5 - \rho_5 K_{65} H_6 = H_{05}, \\ &-\rho K_{46} H_4 - \rho K_{56} H_5 + H_6 = H^*_{06}, \end{aligned}\right\} \tag{8}$$

where

$$H^*_{06} = H_{06} + \tau \left\{ \int_0^{h/w} K_{12}(t)\, H_1(t)\, dt + K_{32} H_3. \right\}$$

Also, from symmetry,

$$K_{45} = K_{46}, \quad K_{54} = K_{64}, \quad \text{and } K_{56} = K_{65}.$$

The determinant of the coefficients is therefore

$$\Delta = \begin{vmatrix} (1 - \rho_4 K_{44}) & -\rho_4 K_{54} & -\rho_4 K_{54} \\ -\rho_5 K_{45} & 1 & -\rho_5 K_{56} \\ -\rho K_{45} & -\rho K_{56} & 1 \end{vmatrix}. \tag{9}$$

Thus

$$H_4 = \frac{1}{\Delta}\Big\{ H_{04}[1 - \rho\rho_5 (K_{56})^2] + \rho_4 H_{05}[\rho K_{54} K_{56} + K_{54}] + \rho_4 H^*_{06}[\rho_5 K_{54} K_{56} + K_{54}] \Big\},$$

$$H_5 = \frac{1}{\Delta}\Big\{ \rho_5 H_{04}[\rho K_{45} K_{56} + K_{45}] + H_{05}[1 - \rho_4 (K_{44} + \rho K_{45} K_{54})] + \rho_5 H^*_{06}[K_{56} + \rho_4 (K_{45} K_{54} - K_{44} K_{56})] \Big\}, \tag{10}$$

$$H_6 = \frac{1}{\Delta}\Big\{ \rho H_{04}[\rho_5 K_{45} K_{56} + K_{45}] + \rho H_{05}[K_{56} + \rho_4 (K_{45} K_{54} - K_{44} K_{56})] + H^*_{06}[1 - \rho_4 (K_{44} + \rho_5 K_{45} K_{54})] \Big\}.$$

Substitution of Eq. 10 in the first three equations of (7) gives

$$H_1(s) = \rho_1 \left\{ \int_0^{h/w} K_{11}(s,t)\, H_1(t)\, dt + K_{21}(s) \cdot H_2 + K_{31}(s) \cdot H_3 \right\},$$

$$H_2 = H_{02}^* + \rho_2^* \left\{ \int_0^{h/w} K_{12}(t)\, H_1(t)\, dt + K_{32} H_3 \right\}, \qquad (11)$$

$$H_3 = \rho_3 \left\{ \int_0^{h/w} K_{13}(t)\, H_1(t)\, dt + K_{23} H_2 \right\}.$$

The helios of the luminous ceiling, with interflections within S_2 but not within the room S_1, is

$$H_{02}^* = \frac{\tau}{\rho\Delta} \Big\{ \rho H_{04}[K_{45} + \rho_5 K_{45} K_{56}] + \rho H_{05}[K_{56} + \rho_4(K_{45}K_{54} - K_{44}K_{56})] + H_{06}[1 - \rho_4(K_{44} + \rho_5 K_{45} K_{54})] \Big\}. \qquad (12)$$

The apparent reflectance of the diaphosor, as seen from below, is

$$\rho_2^* = \rho + \frac{\tau^2}{\rho}\left[\frac{1 - \rho_4(K_{44} + \rho_5 K_{45} K_{54})}{\Delta} - 1\right]. \qquad (13)$$

Note that this effective reflectance is equal to the reflectance ρ of the diaphosor, as measured with a black backing, plus an additional term which depends on the reflectances ρ_4 and ρ_5 of the surfaces above the diaphosor.

Comparison of Eq. 11 with the equations for a single enclosure (3) shows that the two sets of equations are of exactly the same form. The only difference between (11) and the previous equations (for a room with IIa lighting) is that the ceiling helios H_{02} has been replaced by an equivalent helios H_{02}^* and the ceiling reflectance ρ_2 has been replaced by an equivalent diaphosor reflectance ρ_2^*.

In other words, *it is not necessary to solve Eq. 11, since these equations have already been solved and the results tabulated* (3). To make the previous tables directly applicable to the luminous ceiling, we need only calculate the equivalent reflectance ρ_2^* and the helios H_{02}^* (or the logance g). This is the subject of the next section.

5. THE LOGANCE

Calculations are expedited if we express the helios H_{oi} in terms of the pharos from the lamps. Let

F_L = total pharos (lumen) from lamps;
F_4 = incident pharos on walls of enclosure S_2, directly from lamps, without interflections;
F_5 = pharos directly from lamps to surface *5;*
F_6 = pharos directly from lamps to surface *6;*
S = area (m^2) of floor of room;
S_1 = wall area below diaphosor;
S_4 = wall area above diaphosor.

Then

$$H_{04} = \rho_4 \frac{F_4}{S_4}, \qquad H_{05} = \rho_5 \frac{F_5}{S}, \qquad H_{06} = \rho \frac{F_6}{S}.$$

The *domances* or shape factors of the two enclosures are defined as

$$k_r = \frac{S_1}{4S}, \qquad k_r' = \frac{S_4}{4S}$$

Thus,

$$\left.\begin{aligned} H_{04} &= \frac{\rho_4}{4k_r'}\left(\frac{F_4}{F_L}\right)\frac{F_L}{S}, \\ H_{05} &= \rho_5\left(\frac{F_5}{F_L}\right)\frac{F_L}{S}, \\ H_{06} &= \rho\left(\frac{F_6}{F_L}\right)\frac{F_L}{S}. \end{aligned}\right\} \tag{14}$$

Substitution of Eq. 14 in Eq. 12 gives

$$\begin{aligned} H_{02}^* = \frac{F_L\tau}{S\Delta}\Bigg\{ &\frac{\rho_4 K_{45}}{4k_r'}\left(\frac{F_4}{F_L}\right)[1 + \rho_5 K_{56}] \\ &+ \rho_5\left(\frac{F_5}{F_L}\right)[K_{56} + \rho_4(K_{45}K_{54} - K_{44}K_{56})] \\ &+ \left(\frac{F_6}{F_L}\right)[1 - \rho_4(K_{44} + \rho_5 K_{45}K_{54})]\Bigg\}. \end{aligned} \tag{15}$$

We now introduce the logance of the luminous ceiling. Logance may be defined by the equation,

$$g = \frac{H_{02}^* S}{F_L},$$

so

$$\begin{aligned} g = \frac{\tau}{\Delta}\Bigg\{ &\frac{\rho_4 K_{45}}{4k_r'}\left(\frac{F_4}{F_L}\right)[1 + \rho_5 K_{56}] \\ &+ \rho_5\left(\frac{F_5}{F_L}\right)[K_{56} + \rho_4(K_{45}K_{54} - K_{44}K_{56})] \\ &+ \left(\frac{F_6}{F_L}\right)[1 - \rho_4(K_{44} + \rho_5 K_{45}K_{54})]\Bigg\}. \end{aligned} \tag{16}$$

Here, as previously

$$\begin{aligned} \Delta = (1 - \rho_4 K_{44})(1 - \rho\rho_5(K_{56})^2) \\ - \rho_4(\rho + \rho_5)K_{45}K_{54} - 2\rho\rho_4\rho_5 K_{45}K_{54}K_{56}. \end{aligned} \tag{17}$$

6. SPECIAL CASES

In a previous paper (5) the rather complicated expressions for K_{ij} were replaced by exponentials. The same procedure will be employed

here. There are four K's associated with enclosure S_2:

$$K_{55} = e^{-ah'/w} = e^{-2k'_r},$$

$$K_{45} = \int_0^{h'/w} K_{45}(t')\,dt' = a\int_0^{h'/w} e^{-at'}\,dt' = 1 - e^{-2k'_r},$$

$$K_{54} = K_{54}(h'/2w) = \tfrac{1}{2}\,e^{-k'_r},$$

$$K_{44} = K_{44}(h'/2w) = \int_0^{h'/w} K_{44}(h'/2w, t')\,dt'$$

$$= \frac{a}{2}\left\{e^{-ah'/2w}\int_0^{s} e^{at'}\,dt' + e^{ah'/2w}\int_s^{h'/w} e^{-at'}\,dt'\right\} = 1 - e^{-k'_r}.$$

Substitution in Eqs. 13, 16, and 17 results in the equations,

$$\rho_2^* = \rho + \frac{\tau^2}{\rho}\left[\frac{B}{\Delta} - 1\right], \tag{13a}$$

$$g = \frac{\tau}{\Delta}\left\{\frac{\rho_4}{4k'_r}\left(\frac{F_4}{F_L}\right)(1 - e^{-2k'_r})[1 + \rho_5 e^{-2k'_r}] + \frac{\rho_5 e^{-k'_r}}{2}\left(\frac{F_5}{F_L}\right)[2e^{-k'_r} + \rho_4(1 - e^{-k'_r})^2] + B\left(\frac{F_6}{F_L}\right)\right\}, \tag{16a}$$

where

$$\Delta = [1 - \rho_4(1 - e^{-k'_r})][1 - \rho\rho_5 e^{-4k'_r}] - \frac{\rho_4 e^{-k'_r}}{2}(1 - e^{-2k'_r})[\rho + \rho_5(1 + 2\rho e^{-2k'_r})], \tag{17a}$$

and

$$B = 1 - \frac{\rho_4}{2}(1 - e^{-k'_r})[2 + \rho_5 e^{-k'_r}(1 + e^{-k'_r})]. \tag{18}$$

These equations allow the calculation of the equivalent reflectance of the room ceiling, Eq. 13*a*, and the logance of the luminaire, Eq. 16*a*. The helios distribution in the room, and the incident pharosage D_{av} are then found very easily from the published tables(3).

Two special cases will now be considered:

(*A*) The walls of enclosure S_2 reflect negligible light, and
(*B*) The room is very large so that $k'_r \to 0$.

(*A*) Usually the height h' of the lightbox is small compared with the other dimensions so that the light reflected by surface *4* is not of much importance. Suppose it is neglected entirely by letting $\rho_4 = 0$. Then Eq. 18 reduces to

$$\Delta = 1 - \rho\rho_5 e^{-4k'_r}$$

and Eqs. 13*a* and 16*a* become

$$\rho_2^* = \rho + \frac{\tau^2\rho_5 e^{-4k'_r}}{1 - \rho\rho_5 e^{-4k'_r}}, \tag{13b}$$

$$g = \frac{\tau}{1 - \rho\rho_5 e^{-4k'_r}}\left\{\rho_5 e^{-2k'_r}\left(\frac{F_5}{F_L}\right) + \left(\frac{F_6}{F_L}\right)\right\}. \tag{16b}$$

If we assume that half the pharos from the lamps is directly incident on surface *5* and half on surface *6*, Eq. 16*b* becomes

$$g = \frac{\tau}{2(1 - \rho\rho_5 e^{-4k'_r})}\{1 + \rho_5 e^{-2k'_r}\}. \qquad (16c)$$

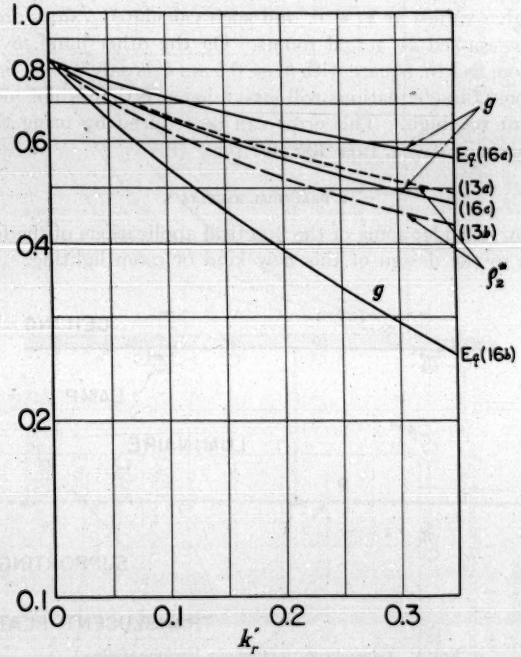


FIG. 3. Calculated curves of logance g and apparent reflectance ρ_2^* for luminous ceilings. Equations 16*a*, 16*b*, 16*c*, 13*a*, and 13*b* were used with $\rho_4 = \rho_5 = 0.90$, $\rho = 0.60$, $\tau = 0.35$ and with the distribution of direct light from the lamps as given in Fig. 5.09, p. 183 of *Lighting Design*.

(*B*) If $k_r' \to 0$, the equations reduce to still simpler forms:

$$\Delta = 1 - \rho\rho_5, \qquad (18a)$$

$$\rho_2^* = \rho + \frac{\tau^2\rho_5}{1 - \rho\rho_5}, \qquad (13c)$$

$$g = \frac{\tau(1 + \rho_5)}{2(1 - \rho\rho_5)}. \qquad (16d)$$

Equations 13*c* and 16*d* are identical with those developed in an entirely different way in another paper (6). In many cases, these very simple formulas provide a satisfactory basis for the design of luminous-ceiling lighting. Figure 3 was calculated by means of the equations given in this section. It shows that both ρ_s^* and g fall off rather rapidly as the lamp enclosure is made higher (k_r' increased). Equations 13*c* and 16*d* give values at $k_r = 0$, and such calculated values are always high when applied to actual rooms. On the other hand, even in as small a room as 5 m. square with $h' = 0.5$ m., k_r' is only 0.10. So in most installations, these equations will give a logance that is not more than 10 per cent too high. This error can be reduced by using the other simple approximations, Eqs. 13*b* and 16*c*.

7. PRACTICAL RESULTS

We now consider some of the practical applications of the foregoing equations to the design of this new kind of room lighting. Figure 4

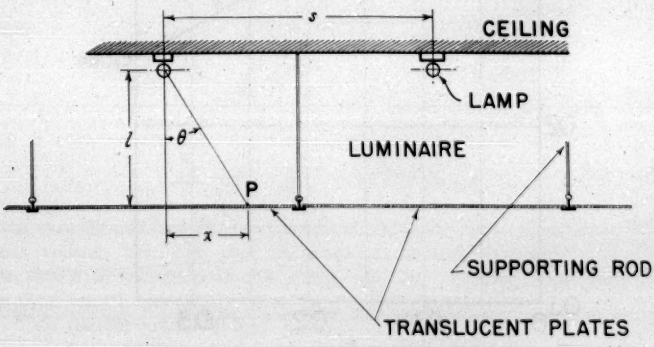


FIG. 4. General construction of a luminous-ceiling.

shows the general type of construction. An investigation of helios uniformity (6) shows that the lamp spacing s should not exceed 3/2 the distance l. The fluorescent lamps are run in continuous rows across the ceiling. Spacing between rows of lamps is s except near the sides of the room (6). The translucent plates may be flat or may be corrugated for greater stiffness (7). Or each section of the diaphosor may consist of a removable frame on which is stretched diffusing paper, thin plastic, or impregnated glass cloth (8). Representative values of ρ and τ are given in Table I.

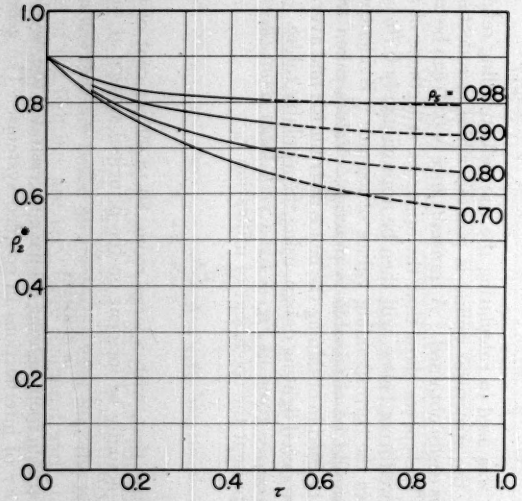


FIG. 5. Apparent reflectance of the luminous ceiling as seen from below. (For $k_r' \to 0$ and $\rho = 0.90 - \tau$.)

ρ_2^* = apparent reflectance of diaphosor,
ρ_s = reflectance of structural ceiling,
τ = transmittance of diaphosor.

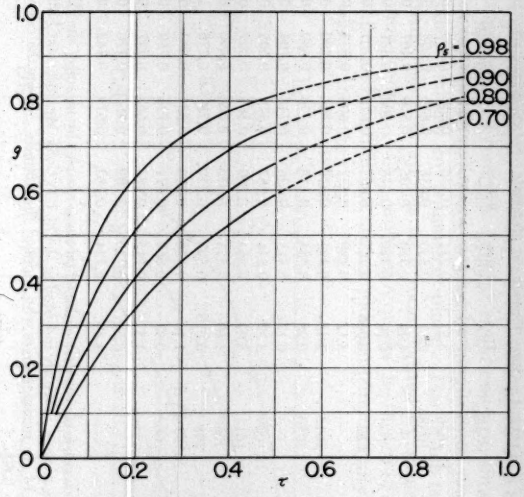


FIG. 6. Equivalent logance ("efficiency") of luminous ceiling. (For $k_r' \to 0$ and $\rho = 0.90 - \tau$.)

$g = \dfrac{\text{lumens from diaphosor}}{\text{lumens from lamps}}$,
ρ_s = reflectance of structural ceiling,
τ = transmittance of diaphosor.

TABLE I.—*Some Translucent Materials.*

Name	Number	Thickness, mm.	Transmittance, τ	Reflectance*, ρ	Calculated† ρ_3^0	g	k_u
Lucite	H 1626	3.57	0.231	0.736	0.840	0.509	0.682
Lucite	H 10588	2.33	0.330	0.638	0.816	0.608	0.801
Flashed opal glass	—	4.44	0.350	0.405	0.550	0.467	0.559
Texloid rayon	—	0.29	0.390	0.545	0.760	0.621	0.805
Texloid paper	682	0.046	0.465	0.502	0.675	0.707	0.886
Plexiglas	—	3.12	0.480	0.352	0.609	0.602	0.737
Lucite	H 1613	3.18	0.491	0.310	0.566	0.590	0.710
Textolite, cloth center	2050	0.98	0.506	0.418	0.726	0.689	0.882
Marlux	—	0.172	0.515	0.426	0.748	0.704	0.906
Lucite	H 1609	3.29	0.534	0.261	0.549	0.608	0.728
Lucite	H 1611	3.07	0.563	0.250	0.569	0.637	0.768
Textolite	2050	1.60	0.571	0.316	0.666	0.690	0.862
Tracing paper	198	0.070	0.598	0.379	0.787	0.776	1.013
Fibestos	20533 TVA	1.16	0.636	0.200	0.584	0.682	0.826
Textolite	2050	0.98	0.660	0.271	0.715	0.759	0.965
Plexiglas•	—	1.52	0.715	0.184	0.663	0.777	0.970
Vinylite	—	0.61	0.720	0.109	0.563	0.710	0.855
Plexiglas	—	1.49	0.724	0.182	0.674	0.762	0.955

NOTE: Spectrophotometric curves were obtained on these samples by means of the Hardy photoelectric spectrophotometer. We are indebted to the M.I.T. Color Measurement Laboratory for the tests and to the manufacturers who supplied the samples. From the curves, ρ and τ were read at $\lambda = 0.60\mu$.

* With black backing.
† Calculated, using the measured values of ρ and τ and for $k_r' = 0$, $\rho_3 = 0.80$. The coefficient of utilization is for an infinite room ($k_r = 0$) with $\rho_3 = 0.30$.
• Exposed 6 months in sunlight.

Equation 13*c* was used in calculating the equivalent ceiling reflectance, Fig. 5. If the structural ceiling is painted white, a reflectance ρ_3 of 0.80 to 0.90 is easily obtained. A transmittance of 0.50 can be used for the plastic. The curves show that the apparent reflectance of the diaphosor, as viewed from below, will then be approximately 0.70.

Figure 6 shows the corresponding values of logance. Evidently, a logance of 0.70 should be obtainable in practice. Values taken from Figs. 5 and 6 can be used in obtaining the helios distribution and average pharosage for a proposed lighting design, using the available tables. As an example, curves are given in Fig. 7 for a very large room ($k_r \to 0$). The ordinates represent the coefficient of utilization k_u:

$$D_{av} = k_u \frac{F_L}{S}.$$

The curves indicate that, with a large room, 80 to 90 per cent of the lamp pharos can actually be incident on the principal surface at table level.

8. SUMMARY

Modern advances in room lighting (9) have necessitated the development of the theory of interflections in coupled systems. The general

case, of two perfectly diffusing enclosures coupled by a translucent plate, is considered. From the general equations are obtained the special cases, (*a*) infinite cylinders coupled by longitudinal plate, and (*b*) finite cylinders coupled by transverse plate. The latter is particularly interesting, since it applies to a room lighted by means of a complete luminous ceiling. This problem requires the theory of coupled systems: the room and the lightbox are not independent, and anything that affects one affects the other also.

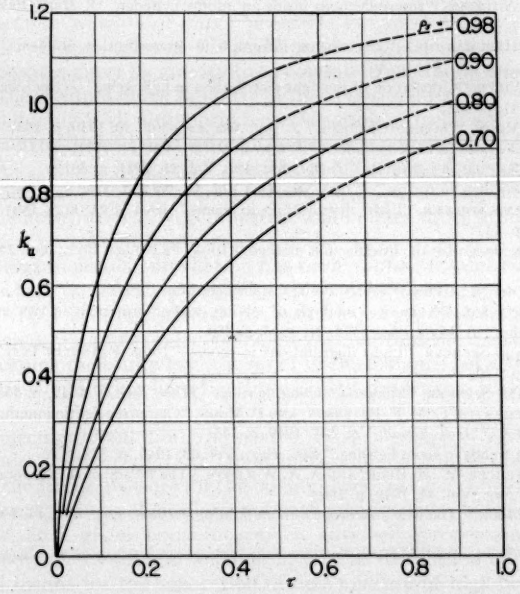


FIG. 7. Coefficient of utilization k_u of an infinite room ($k_r \to 0$, $\rho_s = 0.30$) lighted by a luminous ceiling. The pharosage D_{av} is obtained in terms of k_u by means of the equation,

$$D_{av} = k_u \frac{F_L}{S}$$

where F_L = total pharos (lumen) from lamps,
S = floor area (m^2).

Equations are developed for interflections in cylindrical rooms of any cross-section, with flat ceiling and floor. Particularly simple are the formulas that are based on the assumption that the horizontal dimensions of the lightbox are very large compared with its height. Some representative curves are plotted to aid in the practical design of ceiling lighting.

REFERENCES

(1) H. BUCKLEY, "On the radiation from the inside of a circular cylinder," *Phil. Mag.*, Vol. 4, 1927, p. 753; Vol. 6, 1928, p. 447;
"The blackness of blackbodies and the illumination of lightwells," *J. Opt. Soc. Am.*, Vol. 18, 1929, p. 216.
(2) Z. YAMAUTI, "On the blackness of cylindrical blackbody," *El. Lab., Researches*, Tokyo, No. 378, 1934;
"Etude analytique des 'interréflections' dans un cylindre de longeur infinie," *Comptes rendus Congrès Int. des Appl. l'Eclairage* (Paris, 1937, p. 144); *Rev. Gen. de l'El.*, Vol. 42, 1927, p. 293;
W. F. WHITMORE, "Interreflections inside an infinite cylinder," *J. Math. Phys.*, Vol. 17, 1939, p. 218;
W. R. HEDEMAN, JR., "The cinema integraph in interreflection problems," *J. Math. Phys.*, Vol. 20, 1941, p. 402;
W. T. WHITE, "Calculation of the light distribution in lightwells," *J. Opt. Soc. Am.*, Vol. 31, 1941, p. 308;
PARRY MOON, "On interreflections," *J. Opt. Soc. Am.*, Vol. 30, 1930, p. 195;
"Interreflections in finite cylinders," *J. Opt. Soc. Am.*, Vol. 31, 1941, p. 223;
"Interreflections in lightwells," *J. Opt. Soc. Am.*, Vol. 31, 1941, p. 301;
"Interreflections in rooms," *J. Opt. Soc. Am.*, Vol. 31, 1941, p. 374.
(3) MOON AND SPENCER, "Light distributions in rooms," JOUR. FRANKLIN INST., Vol. 242, 1946, p. 111;
"Lighting design by the interflection method," JOUR. FRANKLIN INST., Vol. 242, 1946, p. 465;
Lighting design (Addison-Wesley Press, Cambridge, Mass., 1948).
(4) P. MAGNUSSON, "Numerical methods of solving integral equations in two independent variables," *J. Math. Phys.*, Vol. 21, 1943, p. 250.
(5) MOON AND SPENCER, "Light distributions in rooms," JOUR. FRANKLIN INST., Vol. 242, 1946, p. 120.
(6) MOON AND SPENCER, "Luminous ceiling lighting," *Illum. Eng.*, Vol. 44, p. 465 (1949).
(7) H. L. BECKWITH, C. M. F. PETERSON, AND P. MOON, "Glare-free lighting methods studied at M.I.T.," *Arch. Record*, Vol. 105, 1949, p. 145;
"Modern trends in room lighting," *Elec. Eng.*, Vol. 68, 1949, p. 577.
(8) D. E. SPENCER, W. H. BUCK, AND A. A. WOLFSON, "The fallacy of the louverall ceiling," *Illum. Eng.*, Vol. 44, 1949, p. 169.
(9) PARRY MOON, "The new approach to room lighting," *Illum. Eng.*, Vol. 44, 1949, p. 221.

NOTES FROM THE NATIONAL BUREAU OF STANDARDS*

MULTICOLUMN REFLUX STILL

A multicolumn countercurrent molecular still designed by Dr. S. L. Madorsky of the National Bureau of Standards appears to have significant advantages for the commercial separation of liquids which differ in their molecular weights or vapor pressures but have similar properties otherwise. The columns of the still [1] are arranged in such a way that the condensates move by gravity from column to column in one direction while all the residues move similarly in the opposite direction. Countercurrent refluxing with automatic recombination of fractions is thus obtained, and the desired separation of fractions is accomplished in a single operation. Among important possible applications of the new device are the concentration of vitamins from natural sources and the purification and concentration of vegetable oils, animal fats, lubricating oils, and high-boiling petroleum fractions.

The term molecular distillation has been applied to that type of distillation where there is no return of escaping molecules to the evaporating surface. This is accomplished by operating with high-boiling (low-vapor-pressure) liquids under such high vacuum that the mean free path of the escaping molecules is of the order of the distance between the surface of the evaporating liquid and the cooled condensing surface of the still.

In molecular distillation, the relative rates of escape of various kinds of molecules from a composite liquid surface are determined by two factors: (1) the vapor pressure (or the boiling point) of each component, and (2) the average molecular velocity of each component. Since molecular velocities, at a given temperature, are inversely proportional to the square roots of the atomic weights, molecular distillation offers a very practical means for the separation of any mixture of high-boiling substances differing in molecular or atomic weight. It is especially useful in the separation of hydrocarbons which do not differ appreciably in boiling point or are unable to withstand the high temperatures of ordinary distillation.

To avoid the necessity of making repeated cuts, with consequent reduction in output, countercurrent molecular stills have been designed in which a number of single-stage stills are so connected that recombination of fractions takes place automatically by gravity feed.[2] Then the

* Communicated by the Director.

[1] For further details, see "A Multicolumn Countercurrent Still," by Samuel L. Madorsky, *J. Research NBS*, Vol. 44, p. 135 (1950) RP 2064.

[2] "Countercurrent Reflux Still for Isotope Separation," *NBS Technical News Bulletin*, Vol. 31, p. 33 (1947).

material to be concentrated can be fed into the system at one end, or at the midpoint if both light and heavy fractions are of interest; and the concentrate can be withdrawn continuously. Countercurrent stills of this type consist of a long column placed at a small angle to the horizontal and divided into cells with an evaporating surface of a few square centimeters of each cell. Such stills have one disadvantage. As the openings between the cells must be relatively large, some mixing of vapor takes place; thus each cell represents merely a subdivision of the column and not an actual single-stage column.

In the multicolumn still developed at the Bureau, each column corresponds to a single cell in the earlier types of countercurrent stills. Mixing of the vapor between stages is thus eliminated, with a corresponding increase in the efficiency of the distillation process. To permit operation on a commercial scale, a large evaporating surface is provided, and the evaporating liquid is spread in a thin film to reduce the hold-up. In addition, the thin film is kept in continuous motion over a rough granular surface; this causes thorough mixing and thus prevents depletion of the lighter constituents.

The Bureau's still consists of ten columns of Pyrex glass, although any number of columns can be used, and they may be made of other materials such as metal. The columns are arranged in a line at successively higher levels and are connected in such a way that the residues or heavy fractions flow from column to column in the downward direction while the condensates or light fractions are pumped from column to column in the upward direction. The over-all operation is such that the light fraction increases in concentration near the lower end.

Each column consists of a central tube, which serves as an evaporator, surrounded by a water-cooled condenser. Below every column is a combination of magnetic pump and reservoir. The liquid is pumped from the reservoir through a feed line into a distributing cup at the top of the column, and from there it spreads over the evaporator in a thin layer. A short ring partition at the lower end of the annular space between the evaporator and condenser forms two concentric annular troughs with a separate outlet from each trough. These troughs and outlets separate the residue flowing down the evaporator from the condensate flowing down the condenser. The residue from each column, except the lowest, flows into the reservoir of the adjacent lower column; and condensate from each column, except the highest, flows into the reservoir of the adjacent higher column. The residue from the lowest column flows back into its own reservoir, while condensate from the highest column flows into a light fraction receiver and from there into the reservoir of the highest column. A large reservoir surrounded by a Nichrome heater serves to degas the liquid before it is introduced into the still. Then, in passing through each feed line the material is preheated to a temperature near that of the evaporator.

The evaporator is 45 cm. long and has an outside diameter of 2.5 cm. This provides an evaporating surface of over 300 sq. cm. To facilitate uniform distribution and mixing of the liquid layer, the evaporating surface is covered with a thin layer of sintered powdered glass. As the condenser tube has an inside diameter of 4.1 cm., the distance between the evaporating and condensing surfaces across the annular space is 0.8 cm.

The magnetic pump at the base of each column consists essentially of a piston fitting closely within the glass reservoir tube and containing an iron core at its upper end. A solenoidal coil surrounds the glass tube in the region of the iron core. When a current flows in the solenoid, the piston moves up and draws liquid from the reservoir through a check valve into a space below the piston. Then when the current is off, the piston drops by gravity, closing the check valve and forcing the liquid through another check valve into the delivery tube.

In a multicolumn still of this type, unless precautions are taken, a gradual accumulation of liquid in some reservoirs and a depletion in others may be expected. This difficulty is avoided in the Bureau's apparatus in two ways: First, the magnetic pumps are constructed as uniformly as possible, and the amplitudes of the pump pistons are equalized. Second, the evaporating rate is adjusted to a little over one-half the rate at which liquid is pumped into the distributing crown of each column, and the excess condensate is allowed to flow by gravity through overflow tubes from column to column in the downward direction. This arrangement also serves to reflux the light fractions from the light reservoir down to the lowest reservoir.

The new multicolumn still should permit large savings of time, labor, and materials since it makes the process of fractionation in molecular distillation a continuous operation. An additional advantage lies in the fact that the entire separation can be carried out without exposing the condensate or residue to the atmosphere; this is particularly desirable for liquids that are susceptible to decomposition upon exposure to air.

RAPID METHOD DETERMINES CURVATURE AND SECOND DERIVATIVE OF A PLOTTED CURVE

A convenient graphical method developed by Dr. H. L. Curtis of the National Bureau of Standards makes possible the quick and accurate determination of the curvature and second derivative at any point on a plotted curve. The method[1] utilizes an easily constructed device known as an "Osculometer," a piece of transparent material containing a series of arcs of known curvature. Curvature is read by direct comparison of the experimental curve with the osculometer, and the corresponding

[1] For further details, see "Determination of Curvature by an Osculometer," by Harvey L. Curtis, *J. Research NBS*, Vol. 44, p. 131 (1950) RP2063.

second derivative is then obtained by a simple calculation. Although the osculometer was originally designed at the Bureau to aid in the analysis of extensive data resulting from a research program on interior ballistics, the method should provide a useful tool in a number of other fields—such as aerodynamics, mechanical engineering, and electrical design—where graphical methods of computation are widely used.

In the Bureau's work on interior ballistics, the acceleration of a projectile was at first determined by plotting two curves on a large scale. Tangents to the first curve, a plot of displacement against time, were obtained at a number of points and were plotted against time to give the second curve, which represented the velocity-time relationship. The tangents to the second curve then gave the instantaneous acceleration as the second derivative of displacement with respect to time. Use of the osculometer was found to simplify the process greatly, eliminating the necessity of plotting the intermediate velocity curve.

In making measurements with the osculometer, a point on the curve is selected at which a value of the second derivative is desired, and an osculometer is chosen having arcs within the proper curvature range. When an arc has been fitted to the curve at the selected point, the curvature, C, is read directly from the osculometer, and the corresponding slope is determined with a protractor by measuring the angle θ between the base of the osculometer and the positive direction of the X-axis. The second derivative of the curve is then computed from the equation

$$\frac{d^2y}{dx^2} = C \sec^3 \theta$$

This equation is easily derived from the familiar expression for radius of curvature.

Several types of osculometers have been developed. The simplest form consists of a transparent sheet of plastic on which black circular lines having uniformly increasing curvature are spaced at regular intervals. A scale at one side gives the curvature of the arcs in reciprocal centimeters. This type of osculometer has the disadvantage that the curve under study is obscured while an arc is being fitted to it, and various methods of avoiding the difficulty have been worked out. For example, the arcs may be made as clear lines in an opaque or semi-opaqu- sheet. The curve to be studied is then drawn on translucent paper, and comparison is made on a diffusing surface illuminated from beneath. Another method is to make each curve a series of dashes. Perhaps the best solution is the double-line osculometer. For each degree of curvature, two identical arcs are drawn about 1 mm. apart. Then the curve under observation can be made to lie between the two arcs having a curvature equal to its own. With such an instrument, curvature can usually be determined to 0.001 reciprocal centimeter, provided the curvature is approximately constant for a distance of 2 cm. on either side of the

point at which it is measured. Thus far, no difference in accuracy has been demonstrated among the various modifications of the instrument.

FREEZING POINT OF URANIUM PRECISELY DETERMINED

The freezing point of uranium, previously known only in a very general way, has been measured[1] to a precision of 2°C. by A. I. Dahl and H. E. Cleaves of the National Bureau of Standards. This high precision was attained by holding the metal at temperatures just above the melting point for several hours, which resulted in a decrease in the content of several impurities and an increase in the freezing temperature to a final constant value.

Until the recent developments in the field of atomic energy, uranium was of little practical importance and was not available in sufficient purity to permit accurate determination of its properties. Thus values for the melting point of uranium given in the literature ranged from 1300° to 3600°C. This wide spread in reported values was largely due to the fact that an oxidized casing or crust is formed on the surface of uranium when it is heated in the presence of most gases. The casing encloses the molten metal until temperatures far above the melting point are reached. Obviously, the true melting point may be much lower than the temperature at which the molten material breaks through the casing.

When a quantity of relatively pure uranium became available as a result of the atomic energy program, the National Bureau of Standards undertook to develop a technique for accurate determination of its freezing point. Oxidation of the sample was prevented by use of a protective atmosphere of purified helium. It was found that when the metal was held just above the melting point for some time, impurities were removed by volatilization and migration to the top and side walls of the container. This procedure resulted in a purity of 99.99 per cent and a consequent rise in the freezing temperature to a constant value of 1133± 2°C.—about 5 deg. above the initial freezing temperature.

In the freezing-point determinations, the uranium was heated about 50° C. above its melting point and then allowed to cool slowly. A hollow enclosure, or black body, was immersed in the uranium, and the freezing temperature was obtained by observing the radiation from the black body with an optical pyrometer during the time the metal was freezing. The uranium was melted in a high-frequency induction furnace. This type of heating caused stirring of the molten sample and thus provided a high degree of temperature uniformity throughout. It also minimized the possibility of contamination by the furnace elements.

[1] For further details, see "The Freezing Point of Uranium," by Andrew I. Dahl and Harold E. Cleaves, *J. Research NBS*, Vol. 43, p. 513 (1949) RP2042.

Three samples of uranium were studied. Two of the samples were from the same extruded rod and had an initial purity of 99.94 per cent. The third was cut from a cast ingot and was about 99.96 per cent pure before heating. Major impurities were carbon and iron.

The three samples were melted in crucibles identical in size and shape, made of beryllia, graphite, and thoria, respectively. Uranium ingots weighing about 380 g. were formed to the shape of the crucible by hot forging, and an axial hole was drilled in each ingot to accommodate the black body. Each sample was put in its crucible and mounted in an insulated porcelain container. This assembly was placed within a Pyrex glass enclosure, which fitted closely inside the water-cooled coils of the induction furnace. The Pyrex enclosure was flushed out with helium for about two hours before heating of the sample was begun, and a helium atmosphere was maintained during the actual heating.

THE FRANKLIN INSTITUTE

MUSEUM

Everyone stops on the street to watch the modern fire engines hurtle by, and nearly all visitors to the Museum pause to regard with an indulgent smile the old fire fighting apparatus which is on display. It is fitting that our Museum should have an exhibit of relics from the early days of fire fighting, because Philadelphia was the first city to organize a fire brigade and the first fire insurance company. As might be expected Benjamin Franklin had a hand in both movements. His newspaper, the *Pennsylvania Gazette*, under date of February 18, 1752, carried a notice to "All persons inclined to subscribe to the articles of insurance of houses from fire," inviting them to attend at the Court House any Saturday prior to April 13th for the purpose of subscribing to the new plan. On April 13th, the subscribers convened and elected the officers and a board of directors.

The first name on the list of subscribers was that of the Lieutenant-Governor of the Province, James Hamilton. The second name on the list was that of Benjamin Franklin. The prominence of his name on this list has led many people to suppose that Franklin was the founder of this organization, but that honor more rightfully belonged to John Smith, who was elected the company's first treasurer.

The name of this first fire insurance company in America was the "Philadelphia Contributorship for the Insurance of Houses from Loss by Fire." The fundamental plan was one of mutual assurance. The policies, printed by Franklin, were for a term of seven years. Later, the liability of the subscriber was limited to the amount of the money he had contributed to obtain his policy. The company was incorporated in 1768, and began to issue perpetual policies in 1810.

The familiar hand-in-hand design on the markers distinguishing homes insured by policy holders in the company was adapted from the design used by an English insurance office, "The Amicable Contributorship and Hand-in-Hand Fire Office." Any contributing member of the Philadelphia company was expected to go to the aid of the fire fighters if summoned to a dwelling marked by the company's distinguishing badge.

The fire fighting equipment displayed in the Museum is headed by an old pumper, dating from about 1735 and believed to be the first of all Philadelphia's fire engines. The later models grew to such size that it is impracticable to exhibit originals and so models of these are shown.

But originals or models, the fire fighting section is well worth a visit since it conveys a comprehensive retrospect of a public service which gets far too little recognition. The city's fire fighting service has always held an enviable reputation throughout the country and a glimpse into its past should aid in refreshing appreciation.

JOURNAL OF THE FRANKLIN INSTITUTE

The following papers will appear in the JOURNAL within the next few months:

BORG, S. F. Additional Interpretations of the Solution of the Straight Beam Differential Equation.

SWANN, W. F. G.: On Certain Matters Pertaining to Electrets.

AYRE, ROBERT S., LYDIK JACOBSEN AND ARIS PHILLIPS: Steady Forced Vibration of a Non-Conservative System with Variable Mass; a Pumping System.

KRZYWOBLOCKI, M. Z.: On the Asymptotic Expansion in Three-Dimensional Compressible Viscous Flow.

KOTTLER, F.: The Distribution of Particle Sizes and the Goodness of Fit.

BLANTON, H. ELMORE: Carrier Compensation for Servomechanisms.

LEE, JAMES: Interflection Tables for Rooms with Floors of High Reflectance.

THE FRANKLIN INSTITUTE

exists today because of the faith and generosity of the men and women who for 125 years have given time and money to its support.

The Institute welcomes financial gifts and bequests and hopes that all those who desire to perpetuate its work will make the The Institute one of their beneficiaries.

FORM OF BEQUEST

*I give and bequeath to The Franklin Institute of the State of Pennsylvania for the Promotion of the Mechanic Arts, the sum of*_____

_____

THE FRANKLIN INSTITUTE LABORATORIES FOR RESEARCH AND DEVELOPMENT

***Abstract of* Stress Distribution Around Spot Welds.**[1]—A. O. BERGHOLM,[2] P. W. SWARTZ,[2] AND G. S. HOELL.[2] In view of the absence of fundamental knowledge relating to the effect of resistance welding on plate joints, a limited research program was approved by the Welding Research Council, and carried out by The Franklin Institute Laboratories for Research and Development.

Frequent failures at or around weld spots, especially where joints were subjected to variable stresses, or reversal of stresses, led to a decision to take a first step toward the determination of factors having significant influence on strength or weakness of a spot weld. Photoelastic models suggested themselves for the purpose of determining the direction of the principal stresses so that strain gages could be placed in these directions. Diagrams were obtained of the isoclinics (Fig. 1) from which the principal stress trajectories have been determined.

In order to corroborate the results found by photoelastic means, several steel specimens were sprayed with Stresscoat Enamel. The results of application of Stresscoat Enamel indicate general conformity with the results obtained from photoelastic models, and these were used as an indication for placing strain gages for actual measurement of magnitude of the stresses.

There were strong indications that the application of a tension load caused a considerably greater stress concentration at the welds than would be generally realized. This was deduced from the fact that there were areas on the outer surfaces of the plates where high compression stresses were clearly indicated. Since under a tension load this cannot take place except from a bending moment, it can be concluded that a very high tension stress occurs near the welds where the two plates are in contact.

[1] Published in *Welding Research Supplement*, Vol. XV, May, 1950, No. 5.

[2] Research Engineers and Consultant on Mechanical Engineering, respectively, The Franklin Institute Laboratories for Research and Development, Benjamin Franklin Parkway at 20th Street, Philadelphia 3, Pa.

NOTE: Reprints of above paper are available in limited quantities. Address: Administration Division, The Franklin Institute Laboratories for Research and Development, Benjamin Franklin Parkway at 20th Street, Philadelphia 3, Pa.

No definite recommendations seem yet to be in order with a view to improving welding design practice or for the purpose of avoiding such stress concentrations as might lead to fatigue failures, without further examination of the stresses due to bending moments set up by eccentric loading. The appearance of an isotropic point existing between the welds (Fig. 1) would indicate that a major change in the stress trajectories would take place in case a hole is drilled at some point between the welds and that this change may very well reduce concentrations. An extension of this research program would include exploration of this phase.

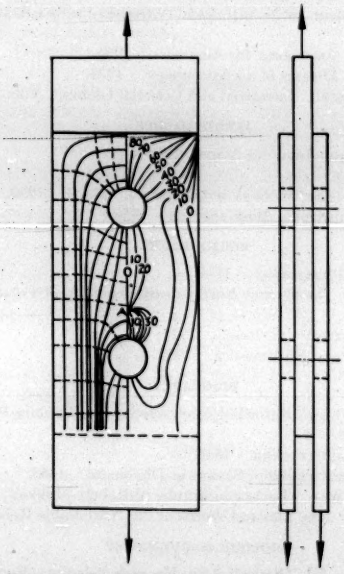


FIG. 1. Isoclinics (shown on the right half) and principal stress trajectories in a two-spot lap-welded specimen. (Point A is an isotropic point.)

These investigations so far lead to a conclusion that single laps of spot-welded joints are inherently weak due to the unavoidable bending of the plates; that in important structures double laps (one plate sandwiched between two plates) will reduce considerably the high stress concentrations set up at the weld spot; that relief of stress concentrations may be obtained by proper removal of material (such as drilled holes) to redistribute the stresses near the weld; and, finally, that the joining of two plates by one row of resistance spot welds is inherently unsound when used for important structures carrying greatly variable loads or reversal of stress.

LIBRARY

The Committee on Library desires to add to the collections any technical works that members would wish to contribute. Contributions will be gratefully acknowledged and placed in the library. Duplicates received will be transferred to other libraries as gifts of the donor.

Photostat Service. Photostat prints of any material in the collections can be supplied on request. The average cost for a print 9 × 14 inches is thirty-five cents.

The Library and reading room are open on Mondays, Tuesdays, Wednesdays and Fridays from 9 A.M. until 5 P.M; Thursdays from 2 P.M. until 10 P.M.; and Saturdays from 9 A.M. until 12 noon.

RECENT ADDITIONS

ARCHITECTURE AND BUILDING

KLEINLOGEL, ADOLF. Influences on Concrete. 1950.

ASTRONOMY

DAVIDSON, MARTIN. Elements of Mathematical Astronomy with a Brief Exposition of Relativity. Ed. 2. 1949.
FLAMMARION, CAMMILLE. Astronomy for Amateurs. 1904.
FLAMMARION, CAMMILLE. Dreams of an Astronomer. 1923.
STEVENSON, EDWARD LUTHER. Terrestrial and Celestial Globes. Vols. 1–2. 1921.

AERODYNAMICS

BIDARD, RENÉ. Thermopropulsion des Avions. 1949.
Guided Missiles. 1948.
JONES, BRADLEY. Elements of Practical Aerodynamics. Ed. 4. 1950.
MILES, EDWARD R. C. Supersonic Aerodynamics. 1950.

BIBLIOGRAPHY

EIGSTI, O. J. Colchicine Bibliography. 1947.
LEEDS AND NORTHRUP CO. Leeds and Northrup Bibliography of Polarographic Literature. 1950.
SCOLARI, FELICE. Alessandro Volta, Roma. 1927.
SMITH, DAVID EUGENE. Rara Arithemetica. 1949.

BIOGRAPHY

COLT, SAMUEL. Sam Colt's own Record of Transactions with Captain Walker and Eli Whitney, Jr. in 1847. 1949.
FARME, CHARLES. Benjamin Franklin. 1846.
LEFEBURE, ALPH. Benjamin Franklin; Savant et Diplomate. 1868.
MILLIKAN, ROBERT ANDREWS. The Autobiography of Robert Millikan. 1950.
MORE, LOUIS TRENCHARD. The Life and Works of the Honourable Robert Boyle. 1944.

BIOLOGICAL CHEMISTRY

FINK, ROBERT MORGAN, JR., Ed. Biological Studies with Polonium, Radium and Plutonium. 1950.
WAKSMAN, SELMAN A. The Actinomycetes. 1950.

CHEMISTRY AND CHEMICAL TECHNOLOGY

ALEXANDER, ELLIOT R. Principles of Ionic Organic Reactions. 1950.
BARRON, HARRY. Modern Plastics. Ed. 2, Rev. 1949.
BRITISH DRUG HOUSES AND HOPKIN AND WILLIAMS, LTD. Analar Standards for Laboratory Chemicals. Ed. 4. 1949.
DAY, ALAN RUSSELL. Electronic Mechanisms of Organic Reactions. 1950.
ELDERFIELD, ROBERT C., Ed. Heterocyclic Compounds. Vol. 1. 1950.
International Colloquium on Macromolecules. 1949. Proceedings. 1950.
KERR, RALPH W. Chemistry and Industry of Starch. Ed. 2. 1950.
KLEINBERG, JACOB. Unfamiliar Oxidation States and Their Stabilization. 1950.
MELLON, MELVIN GUY, Ed. Analytical Adsorption Spectroscopy. 1950.
SIDGWICK, N. V. The Chemical Elements. Two Volumes. 1950.
SISLER, HARRY H. AND STEWART, JAY H. A Systematic Laboratory Course in General Chemistry. 1950.
TIMM, JOHN ARREND. General Chemistry. Ed. 2. 1950.

CIVIL ENGINEERING

BRUCE, ARTHUR G. AND CLARKESON, J. Highway Design and Construction. Ed. 3. 1950.
LEGAT, A. W., DUNN, G. AND FAIRHURST, W. A. Design and Construction of Reinforced Concrete Bridges. 1948.

ECONOMICS

NEWBURY, FRANK D. The American Economic System. 1950.

ELECTRONICS

PENROSE, H. E. Principles and Practice of Radar. 1950.
ROWE, A. P. One Story of Radar. 1948.
WRIGHT, RALPH R. Electronics. 1950.

ENGINEERING

JESSOP, H. T. AND HARRIS, F. C. Photoelasticity. 1950.
MATERIALS HANDLING CONFERENCE. Materials Handling Technique Case Studies Equipment. 1949.
NATIONAL CONFERENCE ON INDUSTRIAL HYDRAULICS. Proceedings. Three Volumes. 1947–1949.
STRAUB, HANS. Die Geschichte der Bauingenieurkunst. 1949.

GENERAL

Author's Guide for Preparing Manuscript and Handling Proof. 1950.
GEBLER, ROBERT T. Philadelphia Ordnance District in World War II. 1949.
MAYO, LAWRENCE SHAW. The Winthrop Family in America. 1948.

GEOLOGY

AMERICAN INSTITUTE OF MINING AND METALLURGICAL ENGINEERS. Industrial Minerals and Rocks. Ed. 2. 1949.
BATEMAN, ALAN MARA. Economic Mineral Deposits. Ed. 2. 1950

HISTORY

CHAPELLE, HOWARD I. The History of the American Sailing Navy. 1949.
HARRISON, H. C. The Story of Sprowston Mill. 1949.
SIMONDS, WILLIAM A. Kamaaina; a Century in Hawaii. 1949.

MANUFACTURE

AMERICAN SOCIETY FOR TESTING MATERIALS. A.S.T.M. Standards on Soaps and Other Detergents. 1947.
JACKSON, CHARLES JAMES. English Goldsmiths and Their Marks. Ed. 2. 1921.
MOSHER, ROBERT H., Ed. Specialty Papers. 1950.

MARINE ENGINEERING

CALDWELL, A. Screw Tug Design. 1946.

MATHEMATICS

BLACK, MAX. The Nature of Mathematics. 1950.
GAY, HAROLD JOSEPH. Analytic Geometry and Calculus. 1950.
KERSHNER, RICHARD BRANDON AND WILCOX, LEE ROY. The Anatomy of Mathematics. 1950.
STEINER, JACOB. Geometrical Constructions with a Ruler Given a Fixed Circle With its Center. 1950.
Symposium in Applied Mathematics. Proceedings. Two Volumes. 1947–1948.

MECHANICAL ENGINEERING

KENT, WILLIAM. Kent's Mechanical Engineers' Handbook. 1950.

METALLURGY

The Failure of Metals by Fatigue. 1950.
HULL, DANIEL R. Casting of Brass and Bronze. 1950.
TEED, P. LITHERLAND. The Properties of Metallic Materials at Low Temperatures. 1950.
UNDERWOOD, L. R. The Rolling of Metals. Vol. 1. 1950.

PHYSICS

BITTER, FRANCIS. Nuclear Physics. 1950.
DINGLE, HERBERT. Practical Applications of Spectrum Analysis. 1950.
ECKERT, E. R. G. Introduction to the Transfer of Heat and Mass. 1950.
HERZBERG, GERHARD. Molecular Spectra and Molecular Structure. Ed. 2. 1950.
International Congress on Rheology. Proceedings. 1948.

RADIO

FRY, D. W. AND GOWARD, F. K. Aerials for Centimetre Wave Lengths. 1950.
GREAT BRITAIN. MINISTRY OF TRANSPORT. Radio Navigation, Radar and Position Fixing Systems for Use in Marine Navigation. 1946.

RAILROADS

ELLIS, CUTHBERT HAMILTON. Some Classic Locomotives. 1949.

SCIENCE

BAWDEN, ARTHUR TALBOT. Man's Physical Universe. Ed. 3. 1950.
HOPKINSON, JOHN. Original Papers. Two Volumes. 1901.
HOPPE, EDMUND. Mathematik und Astronomie im Klassischen Alterum. 1911.

SUGAR

Bagasse Utilization Including Bagasse Patents. 1950.

TRANSPORTATION

HUNTER, LOUIS C. Steamboats on the Western Rivers. 1949.

NOTES FROM THE BIOCHEMICAL RESEARCH FOUNDATION

***Abstract of* Roentgen Irradiation of Desoxyribosenucleic Acid. I. Mechanism of the Action of Irradiation in Aqueous Solution.**—G. LIMPEROS AND W. A. MOSHER (*Am. J. Roentgenol. Radium Therapy*, **63**, 681 (1950)). The study of the mechanism of the action of x-radiation in cellular destruction was undertaken with the thought that if the mechanism were known, it might then be possible to initiate it by some other method in a manner so that only malignant tissues would be destroyed. Growing immature tissue and "radiosensitive" cancers with a higher nucleic acid content than normal had been found to be more easily affected by x-radiation. Because of the importance of desoxyribosenucleic acid (DNA) in cell division, the action of x-radiation on DNA was investigated. The effects of x-radiation on aqueous solutions of DNA were studied to obtain information that might be of value in explaining the effects of irradiation *in vivo*.

DNA prepared from calf thymus was found to be depolymerized in aqueous solution by x-radiation and also by chemically generated free radicals. Thiourea inhibited depolymerization by both means. The extent of the depolymerization was determined by the measurement of the structural viscosity, the streaming birefringence, and the sedimentation constants of the DNA solutions. These results were interpreted as evidence that the mechanism of the action of x-radiation of DNA in aqueous solution is indirect, mediated through the production of free radicals from the solvent.

Experiments in which ethylene glycol was used as a solvent provided evidence of hydrogen bonding in DNA. The inability to depolymerize DNA in a dry state, in ethylene glycol, or in a frozen aqueous solution by x-radiation gave additional evidence for the indirect mechanism.

***Abstract of* Roentgen Irradiation of Desoxyribosenucleic Acid. II. Physicochemical Properties of Desoxyribosenucleic Acid from Irradiated Rats.**—G. LIMPEROS AND W. A. MOSHER (*Am. J. Roentgenol. Radium Therapy*, **63**, 691 (1950)). Physicochemical properties of DNA isolated from irradiated rats were studied to determine whether DNA was depolymerized in the animal and by what mechanism. DNA in irradiated aqueous solutions was shown (Part I) to be depolymerized by an indirect mechanism.

The DNA extracted from the thymus of rats sacrificed immediately after irradiation with 1000 r or 2000 r was found to be low in nitrogen. The purine N/pyrimidine N ratios and the ultraviolet absorption spectra of the DNA solutions revealed that the low nitrogen content was due to a splitting off of purine bases from the DNA.

Complete depolymerization of DNA extracted from the thymus 24 hours after irradiation with 1000 r was indicated by the physicochemical properties of the DNA solutions. Thiourea, injected before irradiation, did not inhibit the splitting off of purine bases from DNA, but did inhibit depolymerization of DNA.

These results suggest that the immediate effect is the result of the direct action of x-radiation while the delayed effect is the result of the indirect action and is possibly due to the generation of free radicals from the cellular water surrounding the DNA.

BOOK REVIEWS

REFLECTIONS OF A PHYSICIST, by Percy Williams Bridgman. 392 pages, 14 × 22 cm. New York, Philosophical Library, 1950. Price, $5.00.

Professor Bridgman's book is divided into five groups of subjects entitled respectively: General Points of View; Applications to Scientific Situations; Primarily Social; Specific Situations; Prophetic.

Most of the chapters are reprints of articles or speeches emanating from the author over a period of some twenty years or more, and they reflect the writer's development of vision in these matters as the history of science has proceeded. Approximately half of the book is devoted to matters primarily of technical scientific interest, in which the so-called operational approach—with which Professor Bridgman's name is so closely associated—is utilized extensively.

The first article under Group I is in part a re-statement of the operational principle and a defense against some of the criticisms which have been raised regarding it.

It must be confessed that those portions of the book which deal with philosophical matters will probably be found hard reading for those who have not, of their own volition, thought deeply about these matters. Quite early in the book, Professor Bridgman voices the situation which must have been experienced by many thoughtful physicists when he says: "As I get older, I lose my sense of *ease* in the operations of logic. I do not think this loss is due to senile decay, but I believe that it is rather because I now see more than I did before." The present reviewer has had this conviction strongly and has the feeling that, if he could forget all that he knew but retained his experience in thinking, he would probably be at the bottom of the class in trying to understand a high school presentation of mechanics. However, it is because the mind develops in this way that points of view which are significant to it in its maturity may tend to leave the layman—or even the unphilosophical physicist—in a state of mind in which he finds difficulty in knowing what is being talked about. As Professor Bridgman implies, the actual understanding is largely a personal affair which the individual must grow within himself, for it is probably not far from the truth to assert that there is no teaching, but only inspiration to learn.

In all philosophic approaches to science one has the problem of drawing the line between what is really relevant for the discussion in the degree of sophistication of the moment and what may be discarded in this degree of sophistication, however much weight it may have when the analysis borders upon wider realms, such as those which question whether there is any universe at all outside of the mind of him who thinks about what he supposes to be the universe. In any discussion, the segregation of the elements into those which are fundamental and those which are trivial at the moment is not always an easy matter and requires one of Professor Bridgman's experience to tread the path with safety and meaningfulness.

The author has some interesting things to say about simplicity when, quoting Poincaré to the effect that any aggregation of phenomena is always susceptible of an infinite number of mechanical implications, he remarks that: "The reason that we are not interested in giving an explanation of quantum phenomena on a purely mechanical basis is that any such explanation would involve the assumption of a prohibitively complicated amount of detail concealed beyond the reach of any contact with experiment. It is natural, therefore, to find that the demand that our theories reproduce reality is becoming replaced by the demand of convenience and simplicity."

Professor Bridgman utters a word of caution against too ready an acceptance of the belief that any particular theory is necessarily final when he writes: "One of the most certain lessons of the past is that no amount of success in the youth of a theory is any guarantee of a hale and hearty old age." This situation is one which is revealed again and again at every stage of the progress of science. In its youth a theory is vital in predicting new correlations, new facts.

In this period the theory says: "Do this, and ye shall find that." Radical in its youth, it becomes conservative with age—conventional in its own radicalness—and having said all that it has to say about the universe, it adopts a negative attitude in saying: "It is useless to do this or that, for to find any important result by doing it would be nonsense in the frame of my philosophy." And so, in the regime of an old theory, discovery seems to have come to an end and science to be dead. Only after some new facts, brought to light in spite of the theory, force a new opening of the inquiry, does knowledge once more proceed to grow.

Naturally, one interested in the field could find on almost every page of the first half of Professor Bridgman's book something on which he could comment, something he would like to enforce still further, something, perhaps, occasionally, with which he would like to disagree, but to attempt any such discussion in a review of the book would make the review longer than the book itself.

Concerning the portion of the book having to do with social affairs and the like, there is probably wide room for discussion, but as the author himself remarks, the vagueness of the meanings attached to words is so great that much of the discussion would probably be of little value unless carried on by a group of people capable of talking on the high plane of sophistication adopted by the author. Unfortunately, most of the people who talk about these things talk on the plane of sophistication where the very imperfections of language provide the means by which they appear to have said something.

There is one matter which Professor Bridgman urges with great courage, that pertaining to his belief that the intelligentia or, in other words, the people capable of thinking clearly, should be privileged by society in relation to the majority of their companions who are essentially stupid. In a democracy, this view will not be found wanting in opponents, who will cry aloud in wrath and use every device of an ambiguous vocabulary to support their contention. However, Professor Bridgman supports his thesis with weighty arguments and however unpalatable the truth may be, it may be one which we shall have to accept.

W. F. G. SWANN

THE AUTOBIOGRAPHY OF ROBERT A. MILLIKAN, 311 pages, plates, 16 × 23 cm. New York, Prentice-Hall, 1950. Price, $4.50.

"The conception of the existence of very minute and very mobile electrical particles, or atoms, goes back nearly two hundred years—indeed to our own Benjamin Franklin. . . ." Indeed, one must return to Franklin's autobiography to find another book as interesting, readable, and human, and at the same time as significant in its breadth of view and in the importance of the events portrayed, as is the present autobiography.

Millikan's active lifetime spans the entire era of modern science, from 1895 when, having earned the doctorate from Columbia University, he arrived in Berlin in time to hear Roentgen's announcement of his early X-ray discoveries, to the present when atomic energy, hardly suspected in 1895, has become one of our major achievements and anxieties.

As one reads this book one has the feeling that Millikan is talking, reading a prepared paper but glancing up every now and then to interject (often with a chuckle) a personal comment that may have just occurred to him, emphasizing a point with his right hand, index finger extended. . He describes his early years in the midwest, his graduate training at Columbia, his years abroad, and his achievements in teaching, writing, and research at Chicago. Then came the first World War, with his contributions to submarine detection and other problems of national defense, and his important part in the organization of the National Research Council. His years at "Caltech," leading to the growth of that outstanding Institute of Technology and to its contributions to the winning of the Second World War, partly in the field of rocket research and development, occupy several chapters.

The chapter entitled "Forks in the Road" describes several instances in which "my good fairy" saw that he was in the right place at the right time. But "I took credit for a few wise decisions myself, and why not, for while the Great Architect had to direct alone the early stages of the revolutionary process . . . it is our sense of responsibility for playing our part to the best of our ability that makes us Godlike."

In the final chapter on "The Two Supreme Elements in Human Progress," Millikan concludes that religion and science, or conscience and knowledge, are the two great forces "which have pulled, and are still pulling, mankind onward and upward." One feels that both forces have been assisted by the author of this autobiography. When he refers to "the matchless scientific achievements of Franklin in the eighteenth century" one is certain that some future writer will use similar language in referring to his work of the twentieth.

C. T. CHASE

THE CHEMISTRY OF INDUSTRIAL TOXICOLOGY, by Hervey B. Elkins. 406 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1950. Price, $5.50.

This book differs greatly from most of the others which have been written on the subject of industrial toxicology. The writers of previous publications in this field have placed principal emphasis on the medical and physiological aspects of the subject. In Dr. Elkins' book the chemical and physical properties of many poisonous substances and the processes in which they are used or produced are discussed in detail.

The first two chapters are introductory in character, dealing with the nature of poisonous substances in general and the types of tests that are used to determine whether or not a hazardous condition exists.

In the next nine chapters specific substances, which either are now or may, in the future, be important in industry are discussed. They are classified under the general headings of: elements, inorganic compounds, organic compounds, natural products and industrial products. The discussion of each compound includes information concerning its physical properties, its harmful effects, the extent to which these effects usually occur, maximum allowable concentration and methods for evaluating the hazards to which workers are exposed.

Chapter 12 deals with preventive measures. Here various methods for minimizing the danger to workers are discussed. Suggestions as to less dangerous materials and processes which may be substituted for some of those now in common use are given.

A discussion of several industrial processes such as: nickel plating, paint spraying, printing and many others is included in the next chapter. In each case the author makes a careful analysis of the hazards involved. Much of this information is presented in tabular form.

In Chapter 14 data on the maximum allowable concentration are given for many specific substances. Chapter 15 is a discussion of certain fallacies and erroneous ideas which have become widely prevalent.

The last two chapters describe equipment and tests used to determine the extent to which toxic substances are present.

At the end of the book the author includes an extensive bibliography to which frequent reference is made throughout the book.

This book treats its subject from a practical standpoint with strong emphasis on preventive measures, and modern techniques for the control and evaluation of hazardous conditions. It should, therefore, be of much value to those responsible for the planning and operation of processes in which toxic substances are involved.

M. D. EARLE

LEGAL PHASES OF ENGINEERING: CONTRACTS AND SPECIFICATIONS, by Ivan C. Crawford. 346 pages, 14 × 22 cm. New York, The Macmillan Co., 1950. Price, $3.75.

This book is prepared primarily as a text book for use in engineering curricula. It deals with the essentials of law with which any practicing engineer should be familiar.

Starting with a general discussion of the importance of a knowledge of the law in relationships between the engineer and his management, contractors, clients, etc., the book leads the reader through an elementary but thorough history of the sources of American law.

The succeeding chapters deal with such important subjects as: evidence, expert testimony, ethics, contracts, business organization, real property, tort, agency, independent contractor, sales contracts, construction contracts, specifications, and insurance.

This book has great value, not only as a text book but as a ready reference for the engineer

to review basic law as it may apply to situations which arise during practice. Each major point covered is further illuminated by case histories, which are well chosen to illustrate the point of law involved.

Four appendixes include typical form contracts, which are very valuable as guides to an engineer who must provide the basic engineering details to a lawyer to enable him to draw a clear contract. While this work concentrates on matters dealing with civil engineering and construction contracts, there is much material that is useful to engineers in other fields.

This text is an extremely valuable addition to a phase of engineering long neglected, and should be included in the active library of all engineers.

J. J. BOERICKE

ADVANCED CHEMICAL CALCULATIONS, by Sylvanus J. Smith. 454 pages, diagrams, 14 × 22 cm. London, The Macmillan Co., 1950. Price, $2.75.

"Advanced Chemical Calculations," is a book designed as a complete course in chemical mathematics suitable for third and fourth year college students. There are twenty one chapters in this volume covering the fields of inorganic, analytical, organic and physical chemistry. The beginning of each chapter explains the theory behind the particular calculation and illustrates several typical problems by solving them step by step. Sets of problems graded as to difficulty are at the end of each chapter. Useful tables containing various data such as logarithms and anti-logarithims, atomic weights, vapor pressures, molal entropies, and others are found in the appendix. A list of the answers to all of the problems follows the appendix.

Smith's objective is to provide practice in chemical calculations with questions varying from the most elementary up to and including those suitable for the honor student. He most certainly accomplishes his purpose and particularly so in the section devoted to physical chemistry. The chapter on the second law of thermo dynamics is to be specifically commended on its completeness and presentation. A knowledge of the calculus will be found necessary for understanding much of the physical chemistry theory.

An objection that could be raised is that the author inserts figures in some of the sample problems without showing their source or giving them dimensional units. Since sample questions are designed to furnish the student with detailed instructions for calculating a particular type of problem, all figures should be carefully explained.

This book together with the author's "Introductory Chemical Calculations," provides a very good and complete course in numerical chemistry. These editions are to be recommended to all students majoring in chemistry.

DONALD H. RUSSELL

ANALYTICAL ABSORPTION SPECTROSCOPY, edited by M. G. Mellon with eight collaborators. 618 pages, tables, drawings and illustrations, 15 × 24 cm. New York, John Wiley & Sons, Inc.; London, Chapman & Hall, Ltd.; 1950. Price, $9.00.

Nine chapters, each contributed by one well qualified in his chosen field, comprise this book, which should prove to be of great value to those engaged in testing and analysis by spectroscopic and allied methods. The authors have included in each chapter enough of general basic principles to insure understanding of the procedures, but the chief emphasis has been placed upon practical problems, and the method and equipment used in analysis in the spectral region 0.2 to 25 microns, i.e. in the ultraviolet, visible and near infrared regions. For those desiring greater detail and a more complete study of any subject, bibliographies, in one case with as many as 574 references, have been appended to each chapter. The material is well organized and clearly presented.

Chapter I deals with the preparation of samples for analysis, reagents available and types of useful reactions. Chapter II defines the terms in use, outlines the general physical principles of measurement of absorption, its nature and the laws governing it.

Chapters III to VIII describe various absorptimeters, their construction and mechanical and optical operation, the advantages of each type of instrument and usual sources of error and the accuracy to be expected. Various factors of importance in absorptimetry such as solvent

used, pH of the solution, concentration, temperature, ionic strength, redox potential, hydrolysis, fluorescence, turbidity etc. are evaluated. Descriptions of available sources and detectors of radiant energy are included. Many specific problems and possible procedures are discussed. An idea of the scope of these chapters is given by their headings which are as follows: Color Comparimeters, Filter Photometers, Spectrophotometers, ultraviolet and visible, Photographic Methods, Applications of Ultraviolet and Visible Spectrophotometric Data, and Spectrophotometers, infrared. The last chapter deals with methods of determining the color of objects by absorptivity. Various systems for the specification of color by matching samples, by spectrophotometric curves and by numbers are explained.

RACHEL FRANKLIN

HANDBOOK OF AERIAL MAPPING AND PHOTOGRAMMETRY, by Lyle G. Trorey. 178 pages, illustrations, plates, 16 × 25 cm. Cambridge University Press, 1950. Price, $5.00.

During the war years the arts of aerial surveys and mapping and photogrammetry saw tremendous development. The former moved from being an auxiliary method to augment ground survey to one of prime importance because of its accuracy and economic features. The subject work is essentially a cartographer's office manual to be used in translating the results of aerial photography to useful maps. The techniques described were devised, in many cases, to solve specific problems of military mapping encountered in the Northwest European Theater in World War II.

Photogrammetry involves the determination of the spatial coordinates of a point by means of two or more photographs containing it. The techniques for this reduction are based upon rules and theorems primarily from perspective geometry.

Following an introduction of the basic theory, methods are described for measuring angles from oblique photographs to determine the apparent and true horizon and azimuth and depression. Succeeding chapters develop the methods of: the perspective grid and the four point method to extend ground and minor control points, the measurement of height from single oblique photographs; stereoscopy, to map from a single overlap of a pair of verticals; contouring; radial line triangulations; and rectification, to correct for tilt. In all cases, little theory is presented, this being introduced in the explanations which accompany the descriptive examples.

The author was responsible for devising and putting into practice variations in many phases of the art of photogrammetry different from the rigid mathematical transformations used previous to the war. These are described with the aid of sketches and worked out examples. Several plotting machines are described, their theory of operation presented, and instructions are given for their use. Included are: the Burns plotter, which plots planimetric detail directly without calculation, from a single oblique of flat country; slotted templates machines, for radial line triangulation; rectification machines; the stereoscopic plotter, which plots a map from a pair of photographs; and the multiplex projectors. The book closes with a review of the standard mapping procedure used by the Royal Canadian Engineers.

The specialized nature of the manual will restrict its use to a small readership. Individuals who are in any way concerned with air survey mapping operations, however, will find it helpful because of the explanation of the technical processes of deriving a finished map or plan from original air photography.

S. CHARP

INTRODUCTION TO THEORETICAL AND EXPERIMENTAL OPTICS, by Joseph Valasek. 454 pages, illustrations, 14 × 22 cm. New York, John Wiley & Sons, Inc., 1949. Price, $6.50.

This book was written primarily as a text to be used in optics courses for advanced undergraduate and first year graduate students. It consists of four parts entitled: geometrical optics, physical optics, radiation and spectra and experimental optics.

In the part on geometrical optics the author includes those topics which can be adequately treated if light is thought of as consisting of independent rays which obey the experimentally observed laws of reflection and refraction. This section is well organized and clearly written. It includes discussions of ideal optical systems and the lens aberrations which place physical

limitations on the formation of images in actual optical systems. Photometry, image brightness and optical apertures are also discussed. Several optical instruments are described in some detail.

Included in the part on physical optics are those phenomena such as diffraction, interference, and polarization which can only be explained in terms of the wave theory of light. There is an interesting section on the optical constants of metals explained in terms of electromagnetic theory.

Line, band and X-ray spectra, thermal radiation, color measurement and light scattering are among the topics discussed in the part on radiation and spectra.

The part on experimental optics contains descriptions of 24 experiments which are well correlated with the material in the text. The discussion of each experiment is written in a clear and concise manner, but the planning of the actual experimental work is left largely to the student.

The book contains 172 problems of varying degrees of difficulty, which help to expand and illustrate the material in the accompanying chapters. The diagrams are carefully drawn and clearly labeled. Throughout the book the author uses bold face type to emphasize important words and phrases. The mathematical treatment in general is quite elementary, but a knowledge of calculus is necessary to follow some of the derivations.

This book should not only prove to be an excellent text for college courses in optics but a valuable reference book as well.

M. D. EARLE

P. H. BRANS' VADE-MECUM 1950 ("THE WORLD'S RADIO TUBES"). Eighth edition, 507 pages, 20 × 29 cm. Antwerp, P. H. Brans, Ltd., 1950.

"Brans' Vade-Mecum 1950" is a volume of characteristics of vacuum tubes manufactured throughout the world. Because of the great number of tubes whose characteristics are given in the tables, the book is somewhat more difficult to use than a manual describing, for instance, only tubes of American manufacture. However, for one who is constantly faced with questions concerning tubes manufactured in all countries, this volume is practically a necessity.

The book is divided into two major parts. In the first half of the book, all tube types are listed in alphabetical and numerical order. Knowing the type number of the tube, the reader refers to this set of tables to find the manufacturer, the number of the table in which the tube's operating characteristics may be found, the heater voltage, and the base number. Other parallel columns in this part of the book list the same data for equivalent tubes, where equivalents exist.

The second half of the book consists of tabular operating data on tubes. Each table in this book bears a number to which the reader is referred upon consulting the front half of the book. In these tables, tubes of the same category are found together. For instance, Table 2.2 contains data on rectifiers, while Table 5.3 contains data on transmitting pentodes, etc. In general, characteristics are given for several conditions of operation.

In the rear of the book are tables of base connections, while the forward part of the volume contains information in five languages on how to make use of the book. This book does not contain any tube characteristic curves. As helpful as the inclusion of such data might be, it would no doubt result in a tremendous book whose cost of preparation would be prohibitive.

R. S. GRUBMEYER

APPLIED SEDIMENTATION, by Parker D. Trask. 707 pages, illustrations, 15 × 24 cm. New York, John Wiley & Sons, Inc., 1950. Price, $5.00.

Sediments are aggregates of solid particles transported by such agencies as wind and water to some place of rest. In "Applied Sedimentation" various problems in the broad field of civil engineering dealing with sediments are presented. Since the subject is so large, involving the effect of sedimentation in soil mechanics, highway engineering, foundations for bridges and buildings, earth dams, tunneling, soil conservation, economic mineral deposits, petroleum and military applications, the various chapters in the book have been written by men who are specialists in each particular field.

The book is divided into seven sections: Basic principles of sedimentation, including sediment transport, origin of soils and associated geophysical problems; Engineering problems involving strength of sediments, including foundation problems, earth dams and soft ground tunneling; Applications of processes of sedimentation, including landslides, permafrost, sedimentation in harbors and reservoirs, and debris control; Applications involving nature of constituents, including influence of sedimentation on concrete aggregate, clays, and foundry sands; Economic mineral deposits, including sedimentary mineral deposits and geochemical prospecting for ores; Petroleum geology problems, including subsurface techniques, porosity, permeability and capillarity of petroleum reservoirs; and Military applications, including sedimentary materials in military geology and applications of sedimentation to naval problems.

Broadly speaking, the articles, or chapters, are quite general in nature. If the reader expects to find answers to particular problems in sedimentation, as suggested by the fly leaf designation of "handbook," he will be disappointed. He will find, usually, a general description of the particular subject, including an introduction, a broad discussion of the problems involved (depending upon the subject), the current state of progress, suggested research to advance the state of knowledge, and an extensive bibliography. Although "Applied Sedimentation" omits many specific answers an engineer might require, the copious references should supply him with ample information.

WALTER HAMMER, JR.

ELEMENTS OF HEAT TRANSFER AND INSULATION, by Max Jakob and G. A. Hawkins. Second edition, 230 pages, illustrations, 15 × 23 cm. New York. John Wiley & Sons Inc., 1950. Price, $4.00.

This book is a revision of one by the same authors entitled "Elements of Heat Transfer" which was published in 1942. It is intended to be used as a text in an elementary course in heat transfer for undergraduate engineering students. The book covers the subject of heat transfer quite thoroughly. In addition to chapters on dimensional analysis, conduction of heat in the steady state and conduction under non-equilibrium conditions, there are also chapters on the transfer of heat by each of the following methods: free convection, forced convection, condensing and boiling, radiation, and by various combinations of the above methods. There are also chapters on the experimental determination of conductivities and emissivities and on fluid friction as related to heat transfer.

The chief changes which have been made by the authors in preparing the revised edition have been the expansion of several of the chapters to include more detail, the addition of several tables and the insertion of some graphical methods for the solution of heat transfer problems. The number of problems has been increased from 83 to 220. The number of chapters and their arrangement are the same as in the first edition.

The pattern adopted by the authors throughout the book is to follow the discussion of a topic by one or more specific examples for which complete solutions are given. The mathematical treatment is not difficult although some knowledge of calculus and differential equations is necessary.

The new edition of this book should be an improvement over the first since the authors have expanded the book where their teaching experience has shown it to be necessary.

M. D. EARLE

DIE GRUNDLAGEN DER ARITHMETIK, by G. Frege. English translation by J. L. Austin. 119 pages, 14 × 22 cm. Breslau, Verlag von Wilhelm Koebner, 1884, and New York, Philosophical Library, 1950. Price, $4.75.

This book is noteworthy on several counts: (1) It contains what appears to be a facsimile reproduction of the original German edition of Frege's "logico-mathematical enquiry into the concept of number," which stands at the dawn of development of the modern theory of numbers; (2) It includes a carefully prepared English translation of the original text for those readers who are averse or unequal to the task of handling the intricacies and idioms of the sister tongue;

and (3) Better yet, the original and the translated texts are arranged, paragraph by paragraph, on opposing pages of the book, so that a direct comparison of both is possible in the most convenient manner.

The present reviewer, who is fully conversant with German, has repeatedly checked the accuracy of Mr. Austin's translation, by preparing his own English version of a sentence or two at a time in various parts of the book and comparing it with his. In all instances Mr. Austin's version was either identical with that of this reviewer or superior to it in conveying the logical niceties of Frege's arguments.

The necessity, in a text of this nature, of rendering, as it were, verbatim, the rational content of the author's statements in German makes it unavoidable that the English version becomes somewhat stilted. Unavoidably too, much of the original style and mood has been lost.

Despite its formidable subject, Frege's original text makes delightful reading. The first half of the book is devoted to the presentation of the views of other writers on the nature of arithmetical propositions, on the concept of Number, and on those of Unity and One. Systematically, Frege takes Kant, Leibniz, Hankel, Mills, Hobbes, Locke and a host of less illustrious predecessors to task, quoting their statements on the subject and neatly plucking them apart with a wry smile. Mills, in particular, as a proponent of empiricism, comes in for a large share of genteel sarcasm.

The second half of the book contains demonstrations of Frege's thesis that "a statement of number contains an assertion about a concept," so that the laws of arithmetic are analytic judgments and, consequently, *a priori*.

In the concluding section the statement occurs: "Arithmetic is but an extension of logic and every proposition of arithmetic is a law of logic, albeit a derivative one. To apply arithmetic in the physical sciences is to bring logic to bear on observed facts (observation itself already includes within it a logical activity); calculation is deduction." A thoroughly modern point of view!

OTTO R. SPIES

BOOK NOTES

A COURSE IN GENERAL CHEMISTRY, SEMI-MICRO ALTERNATE FORM, by William C. Bray, W. M. Latimer and R. E. Powell. Third edition, 217 pages, illustrations, 14 × 22 cm. New York, The Macmillan Co., 1950. Price, $3.00.

To facilitate the use of centrifugation with its advantages, the authors have rewritten their textbook on general inorganic chemistry employing semi-micro techniques. Other changes in instructional methods have been incorporated and a greater use has been made of the pictorial representation of laboratory techniques.

GENERAL CHEMISTRY, by John A. Timm. Second edition, 764 pages, illustrations, 16 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $4.50.

This first-year college chemistry text has been revised to place greater emphasis on the descriptive chemistry of the elements and their compounds. Much new material has also been added on such topics as the electronegativity scale, resonance, and the structure of silica. The sections dealing with nucleonics have been brought up to date in line with the recent developments.

MAN'S PHYSICAL UNIVERSE, by Arthur Talbot Cawden. Third edition, 822 pages, illustrations, 14 × 22 cm. New York, The Macmillan Co., 1950. Price, $4.75.

This college survey text in physical science shows no major changes in treatment from the previous edition. New matter has been added to bring it up to date and a few chapters have been rather extensively rewritten. The bibliography has been omitted but a mimeographed list of references is available from the author.

ANALYTIC GEOMETRY AND CALCULUS, by Harold J. Gay, edited by Raymond K. Morley. 524 pages, diagrams, 15 × 23 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $5.00.

A first course textbook in analytic geometry and calculus which correlates both subjects in such a way that the applications of each to the other are considered. Plane analytic geometry and differential and integral calculus are considered in the first half of the book, and in the second half, solid analytic geometry and calculus are added as well as elementary differential equations. The treatment is in part by kinds of equations. Numerous illustrative examples and exercises have been included.

ECONOMIC MINERAL DEPOSITS, by Alan M. Bateman. Second edition, 916 pages, illustrations, 16 × 24 cm. New York, John Wiley & Sons, Inc., 1950. Price, $7.50.

Designed primarily as a textbook, this work has found use as a source of information for all those interested in mineral deposits. The general scope of the first edition has been retained, although rather extensive changes have been made in Part I dealing with the principles and processes of the formation of mineral deposits. The lists of selected references have been considerably expanded. The broad scope of the volume with its detailed treatment of foreign deposits as well as American, contributes greatly to its practicality and usefulness.

KENTS' MECHANICAL ENGINEERS' HANDBOOK, in two volumes. POWER VOLUME, edited by J. Kenneth Salisbury; DESIGN AND PRODUCTION VOLUME, edited by Colin Carmichael. Twelfth edition, illustrations, 15 × 22 cm. New York, John Wiley & Sons, Inc., 1950. Price, $8.50 each volume.

Any work which has been published as long (fifty-five years) as Kents' handbook needs little introduction to its potential users. This note, then, need record only the fact that a new edition has appeared and point out the major changes that have been made. Each volume has undergone considerable rearrangement of the order of sections, with both additions and deletions. In the volume on power, considerable emphasis has been placed on the new developments such as atomic power, combustion gas turbines, supersonics, jet propulsion, heat pumps, panel heating and others.

The volume on design bears a new subtitle *Design and Production* in place of the previous *Design, Shop Practice*, giving recognition to "the fact that design and production are becoming recognized as simply two phases of a single activity—the engineering of a product from the inception of an idea to the completion of the finished article at an economical cost." Both volumes show a large increase in the number of pages, and both have detailed indexes. Based on the contributions of a large number of authorities, Kents' in the new edition will continue to merit the esteem in which it is held.

SEMI-CONDUCTEURS ÉLECTRONIQUES ET COMPLEXES DÉRIVÉS, THÉORIES—APPLICATIONS, by Stanislas Teszner. 96 pages, illustrations, 21 × 30 cm. Paris, Gauthier-Villars, 1950. Price, 1200 fr.

In this work on electronic semi-conductors, the author treats, in the first part, of the various phenomena of these substances and discusses the theories which have been advanced to explain them. In the second part he considers the practical applications of the semi-conductors, such as rectifiers, detectors and thermistors. With a lengthy bibilography, this should prove a useful summary of both the scientific and technical information on a subject of increasing importance in electricity and radio.

STRENGTH OF MATERIALS, by James E. Boyd. Fifth edition, revised by Samuel B. Folk. 417 pages, illustrations, diagrams, 15 × 24 cm. New York, McGraw-Hill Book Co., Inc., 1950. Price, $4.25.

A standard textbook on a basic phase of engineering now appears in a fifth edition in which the editor has made a conscious attempt to emphasize the fundamentals of the theory. Various changes have been made in the order of topics; Mohr's circle for the solution of simple stresses has been introduced early to give the student familiarity with its use, and new material has been included. Problems have been added to bring the total to over a thousand.

CURRENT TOPICS

Electrical Directional Signals.—Following the lead of other states, the New York legislature has just passed a bill that will undoubtedly lead to reduced wear and tear on automobile horns and brake linings and drivers' tempers. Far more important, it may well help check the rising accident rate.

The new measure requires the installation of electrical directional signals on every motor vehicle manufactured for use in New York State after January 1, 1952. These inexpensive direction indicators so effectively cut down the hazards of turning in traffic that police and other specialists in highway safety are beginning to rank them with stop lights and rear vision mirrors as "musts" for really safe driving.

So may late-model automobiles are equipped with these winking arrows at front and rear that most drivers are already familiar with them. However, few know the remarkably ingenious but fool-proof principle that makes them work. Into an ordinary circuit is coupled a flasher relay, or current intermittor. In the type of intermittor made by Tung-Sol Lamp Works, Inc., Newark 4, N. J., the flow of current is rapidly turned on and off by a spring-biased armature actuated by a length of Nichrome* resistance wire.

When the driver flicks the hand switch to indicate a turn, current flows through the Nichrome wire, heating it up until thermal expansion lengthens it. The spring-biased armature then snaps shut a pair of contacts so that the current bypasses the Nichrome wire, lighting the directional arrows. The wire cools and contracts, until it snaps the contacts open again and the lights wink off. All this occurs in a second or less; various Tung-Sol intermittors have flashing rates of from 40 to 180 times per minute.

According to Tung-Sol, Nichrome turned out to be the ideal material for this key actuating part of their intermittors. They report: "It has high tensile strength under elevated temperature conditions. It has high specific electrical resistance. It has a high temperature coefficient of linear expansion. At the same time, it is corrosion resistant and does not oxidize under elevated temperatures. It can be welded. It has good stability, including low creep, and is fatigue resistant."

Precision Scale for Architects and Engineers (*Bakelite Review*, Vol. 22, No. 1).—A precision triangular scale for architects and engineers made of Vinylite rigid plastic sheet strips gripped by an aluminum core gives far longer wear than conventional scales at less than one-seventh the price. Notable for its dimensional stability, this plastic is also resistant to water, oil, grease, and most chemicals. Neither it nor the aluminum core is subject to warping, and the fins and core are interlocked by a special process which allows for the different expansion coefficients of the two materials. Markings on white edges of the scale fins are also applied by a special process, then coated with a Vinylite resin base liquid so they will withstand continuous wear without rubbing off.

* Trade Mark Reg. U. S. Pat. Off.